

Nonstrange baryonia

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Abstract

The relativistic six-quark equations including the u , d quarks and antiquarks are found. The nonstrange baryonia $B\bar{B}$ are constructed without the mixing of the quarks and antiquarks. The relativistic six-quark amplitudes of the baryonia are calculated. The poles of these amplitudes determine the masses of baryonia. 15 masses of baryonia are predicted. The mass of baryonium with the spin-parity $J^P = 0^-$ $M = 1835 \text{ MeV}$ is used as a fit.

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I. Introduction.

BES Collaboration observed a significant threshold enhancement of $p\bar{p}$ mass spectrum in the radiative decay $J/\psi \rightarrow \gamma p\bar{p}$ [1]. Recently BES Collaboration reported the results on $X(1835)$ in the $J/\psi \rightarrow \gamma \eta' \pi^+ \pi^-$ channel [2]. Under the strong assumption that the $p\bar{p}$ threshold enhancement and $X(1835)$ are the same resonance, Zhu and Gao suggested $X(1835)$ could be a $J^{PC} = 0^{-+}$ $I^G = 0^+$ $p\bar{p}$ baryonium [3].

Theoretical investigations of baryon-antibaryon bound states date back to the proposal of Fermi and Yang [4] to make the pion out of a nucleon-antinucleon pair. The model of Nambu and Jona-Lasinio [5] which is constructed to give a nearly zero-mass pion as a fermion-antifermion bound state, also has a scalar resonance of twice the fermion mass. Enhancement in the baryon-antibaryon channel near the threshold are expected on the basis of duality arguments [6 – 8] and by comparison with the systematic of resonance formation in meson-meson and meson-baryon channels [9]. A historical survey of bound states or resonances coupled to the nucleon-antinucleon channel is given in Ref. [10]. Gluonic states can couple to baryon-antibaryon channels of appropriate spin and parity. The discussions of B decays involving baryon-antibaryon pairs include Refs. [11 – 15].

Theoretical work speculated many possibilities for the enhancement such as the t-channel pion exchange, some kind of threshold kinematical effects, as new resonance below threshold or $p\bar{p}$ bound state [16 – 23].

In a series of papers [24 – 28] a method has been developed which is convenient for analysing relativistic three-hadron systems. The physics of the three-hadron system can be described by means of a pair interaction between the particles. There are three isobar channels, each of which consists of a two-particle isobar and the third particle. The presence of the isobar representation together with the condition of unitarity in the pair energies and of analyticity leads to a system of integral equations in a single variable. Their solution makes it possible to describe the interaction of the produced particles in three-hadron systems.

In Ref. [29] a representation of the Faddeev equation in the form of a dispersion relation in the pair energy of the two interacting particles was used. This was found to be convenient in order to obtain an approximate solution of the Faddeev equation by a method based on extraction of the leading singularities of the amplitude. With a rather crude approximation of the low-energy NN interaction a relatively good description of the form factor of tritium (helium-3) at low q^2 was obtained.

In our papers [30 – 32] relativistic generalization of the three-body Faddeev equations was obtained in the form of dispersion relations in the pair energy of two interacting quarks. The mass spectrum of S -wave baryons including u , d , s quarks was calculated by a method based on isolating the leading singularities in the amplitude. We searched for the approximate solution of integral three-quark equations by taking into account two-particle and triangle singularities, all the weaker ones being neglected. If we considered such approximation, which corresponds to taking into account two-body and triangle singularities, and defined all the smooth functions of the subenergy variables (as compared with the singular part of the amplitude) in the middle point of the physical region of Dalitz-plot, then the problem was reduced to the one of solving a system of simple algebraic equations.

In the previous paper [35] the relativistic six-quark equations are found in the framework of coupled-channel formalism. The dynamical mixing between the subamplitudes of hexaquark are considered. The six-quark amplitudes of dibaryons are calculated. The poles of these amplitudes determine the masses of dibaryons. We calculated the contribution of six-quark subamplitudes to the hexaquark amplitudes.

In the present paper the relativistic six-quark equations including u , d quarks and antiquarks are found. The nonstrange barionia $B\bar{B}$ are constructed without the mixing of the quarks and antiquarks. The relativistic six-quark amplitudes of the barionia are calculated. The poles of these amplitudes determine the masses of barionia. In Sec. II we briefly discuss the relativistic Faddeev approach. The relativistic three-quark equations are constructed in the form of the dispersion relations over the two-body subenergy. The approximate solution of these equations using the method based on the extraction of leading singularities of the amplitude are obtained. We calculated the mass spectrum of S -wave baryons with $J^P = \frac{1}{2}^+, \frac{3}{2}^+$ (Table I). In Sec. III the six-quark amplitudes of barionia are constructed. The dynamical mixing between the subamplitudes of barionia are considered. The relativistic six-quark equations are obtained in the form of the dispersion relations over the two-body subenergy. The approximate solutions of these equations using the method based on the extraction of leading singularities of the amplitude are obtained. Sec. IV is devoted to the calculation results for the barionia mass spectrum and the contributions of subamplitudes to the barionia amplitude (Tables II, III, IV). In conclusion, the status of the considered model is discussed.

II. Brief introduction of relativistic Faddeev equations.

We consider the derivation of the relativistic generalization of the Faddeev equation for the example of the Δ -isobar ($J^P = \frac{3}{2}^+$). This is convenient because the spin-flavour part of the wave function of the Δ -isobar contains only nonstrange quarks and pair interactions with the quantum numbers of a $J^P = 1^+$ diquark (in the color state $\bar{3}_c$). The $3q$ baryon state Δ is constructed as color singlet. Suppose that there is a Δ -isobar current which produces three u quarks (Fig. 1a). Successive pair interactions lead to the diagrams shown in Fig. 1b-1f. These diagrams can be grouped according to which of the three quark pairs undergoes the last interaction i.e., the total amplitude can be represented as a sum of diagrams. Taking into account the equality of all pair interactions of nonstrange quarks in the state with $J^P = 1^+$, we obtain the corresponding equation for the amplitudes:

$$A_1(s, s_{12}, s_{13}, s_{23}) = \lambda + A_1(s, s_{12}) + A_1(s, s_{13}) + A_1(s, s_{23}). \quad (1)$$

Here, the s_{ik} are the pair energies of particles 1, 2 and 3, and s is the total energy of the system. Using the diagrams of Fig. 1, it is easy to write down a graphical equation for the function $A_1(s, s_{12})$ (Fig. 2). To write down a concrete equations for the function $A_1(s, s_{12})$ we must specify the amplitude of the pair interaction of the quarks. We write the amplitude of the interaction of two quarks in the state $J^P = 1^+$ in the form:

$$a_1(s_{12}) = \frac{G_1^2(s_{12})}{1 - B_1(s_{12})}, \quad (2)$$

$$B_1(s_{12}) = \int_{4m^2}^{\infty} \frac{ds'_{12}}{\pi} \frac{\rho_1(s'_{12}) G_1^2(s'_{12})}{s'_{12} - s_{12}}, \quad (3)$$

$$\rho_1(s_{12}) = \left(\frac{1}{3} \frac{s_{12}}{4m^2} + \frac{1}{6} \right) \left(\frac{s_{12} - 4m^2}{s_{12}} \right)^{\frac{1}{2}}. \quad (4)$$

Here $G_1(s_{12})$ is the vertex function of a diquark with $J^P = 1^+$. $B_1(s_{12})$ is the Chew-Mandelstam function [34], and $\rho_1(s_{12})$ is the phase spaces for a diquark with $J^P = 1^+$.

The pair quarks amplitudes $qq \rightarrow qq$ are calculated in the framework of the dispersion N/D method with the input four-fermion interaction [35, 36] with the quantum numbers of the gluon [37, 38].

The four-quark interaction is considered as an input:

$$g_V (\bar{q} \lambda I_f \gamma_\mu q)^2 + 2 g_V^{(s)} (\bar{q} \lambda I_f \gamma_\mu q) (\bar{s} \lambda \gamma_\mu s) + g_V^{(ss)} (\bar{s} \lambda \gamma_\mu s)^2. \quad (5)$$

Here I_f is the unity matrix in the flavor space (u, d). λ are the color Gell-Mann matrices. Dimensional constants of the four-fermion interaction g_V , $g_V^{(s)}$ and $g_V^{(ss)}$ are parameters of the model. At $g_V = g_V^{(s)} = g_V^{(ss)}$ the flavor $SU(3)_f$ symmetry occurs. The strange quark violates the flavor $SU(3)_f$ symmetry. In order to avoid additional violation parameters we introduce the scale of the dimensional parameters [38]:

$$g = \frac{m^2}{\pi^2} g_V = \frac{(m + m_s)^2}{4\pi^2} g_V^{(s)} = \frac{m_s^2}{\pi^2} g_V^{(ss)}, \quad (6)$$

$$\Lambda = \frac{4\Lambda(ik)}{(m_i + m_k)^2}.$$

Here m_i and m_k are the quark masses in the intermediate state of the quark loop. Dimensionless parameters g and Λ are supposed to be constants which are independent of the quark interaction type. The applicability of Eq. (5) is verified by the success of De Rujula-Georgi-Glashow quark model [37], where only the short-range part of Breit potential connected with the gluon exchange is responsible for the mass splitting in hadron multiplets.

In the case under discussion the interacting pairs of particles do not form bound states. Therefore, the integration in the dispersion integral (7) run from $4m^2$ to ∞ . The equation corresponding to Fig. 2 can be written in the form:

$$\begin{aligned}
A_1(s, s_{12}) &= \frac{\lambda_1 B_1(s_{12})}{1 - B_1(s_{12})} + \frac{G_1(s_{12})}{1 - B_1(s_{12})} \int_{4m^2}^{\infty} \frac{ds'_{12}}{\pi} \frac{\rho_1(s'_{12})}{s'_{12} - s_{12}} G_1(s'_{12}) \\
&\times \int_{-1}^{+1} \frac{dz}{2} [A_1(s, s'_{13}) + A_1(s, s'_{23})]. \tag{7}
\end{aligned}$$

In Eq. (7) z is the cosine of the angle between the relative momentum of particles 1 and 2 in the intermediate state and the momentum of the third particle in the final state in the c.m.s. of the particles 1 and 2. In our case of equal mass of the quarks 1, 2 and 3, s'_{13} and s'_{12} are related by the equation (8) (See Ref. [39])

$$s'_{13} = 2m^2 - \frac{(s'_{12} + m^2 - s)}{2} \pm \frac{z}{2} \sqrt{\frac{(s'_{12} - 4m^2)}{s'_{12}} (s'_{12} - (\sqrt{s} + m)^2)(s'_{12} - (\sqrt{s} - m)^2)}. \tag{8}$$

The expression for s'_{23} is similar to (8) with the replacement $z \rightarrow -z$. This makes it possible to replace $[A_1(s, s'_{13}) + A_1(s, s'_{23})]$ in (7) by $2A_1(s, s'_{13})$.

From the amplitude $A_1(s, s_{12})$ we shall extract the singularities of the diquark amplitude:

$$A_1(s, s_{12}) = \frac{\alpha_1(s, s_{12}) B_1(s_{12})}{1 - B_1(s_{12})}. \tag{9}$$

The equation for the reduced amplitude $\alpha_1(s, s_{12})$ can be written as

$$\alpha_1(s, s_{12}) = \lambda + \frac{1}{B_1(s_{12})} \int_{4m^2}^{\infty} \frac{ds'_{12}}{\pi} \frac{\rho_1(s'_{12})}{s'_{12} - s_{12}} G_1(s'_{12}) \int_{-1}^{+1} \frac{dz}{2} \frac{2\alpha_1(s, s'_{13}) B_1(s'_{13})}{1 - B_1(s'_{13})}. \tag{10}$$

The next step is to include into (10) a cutoff at large s'_{12} . This cutoff is needed to approximate the contribution of the interaction at short distances. In this connection we shall rewrite Eq. (10) as

$$\alpha_1(s, s_{12}) = \lambda + \frac{1}{B_1(s_{12})} \int_{4m^2}^{\infty} \frac{ds'_{12}}{\pi} \Theta(\Lambda - s'_{12}) \frac{\rho_1(s'_{12})}{s'_{12} - s_{12}} G_1(s'_{12}) \int_{-1}^{+1} \frac{dz}{2} \frac{2\alpha_1(s, s'_{13}) B_1(s'_{13})}{1 - B_1(s'_{13})}. \tag{11}$$

In Eq. (11) we have chosen a hard cutoff. However, we can also use a soft cutoff, for instance $G_1(s'_{12}) = G_1 \exp\left(-\frac{(s'_{12} - 4m^2)^2}{\Lambda^2}\right)$, which leaves the results of calculations of the mass spectrum essentially unchanged.

The construction of the approximate solution of Eq. (11) is based on extraction of the leading singularities are close to the region $s_{ik} \approx 4m^2$. The structure of the singularities of amplitudes with a different number of rescattering (Fig. 1) is the following [39]. The strongest singularities in s_{ik} arise from pair rescatterings of quarks: square-root singularity corresponding to a threshold and pole singularities corresponding to bound states (on the first sheet in the case of real bound states, and on the second sheet in the case of virtual bound states). The diagrams of Figs. 1b and 1c have only these two-particle singularities. In addition to two-particle singularities diagrams of Figs. 1d and 1e have their own specific triangle singularities. The diagram of Figs. 1f describes a

larger number of three-particle singularities. In addition to singularities of triangle type it contains other weaker singularities. Such a classification of singularities makes it possible to search for an approximate solution of Eq. (11), taking into account a definite number of leading singularities and neglecting the weaker ones. We use the approximation in which the singularity corresponding to a single interaction of all three particles, the triangle singularity, is taken into account.

For fixed values of s and s'_{12} the integration is carried out over the region of the variable s'_{13} corresponding to a physical transition of the current into three quarks (the physical region of Dalitz plot). It is convenient to take the central point of this region, corresponding to $z = 0$, to determinate the function $\alpha_1(s, s_{12})$ and also the Chew-Mandelstam function $B_1(s_{12})$ at the point $s_{12} = s_0 = \frac{s}{3} + m^2$. Then the equation for the Δ isobar takes the form:

$$\alpha_1(s, s_0) = \lambda + I_{1,1}(s, s_0) \cdot 2 \alpha_1(s, s_0), \quad (12)$$

$$I_{1,1}(s, s_0) = \int_{4m^2}^{\Lambda_1} \frac{ds'_{12}}{\pi} \frac{\rho_1(s'_{12})}{s'_{12} - s_{12}} G_1 \int_{-1}^{+1} \frac{dz}{2} \frac{G_1}{1 - B_1(s'_{13})}. \quad (13)$$

We can obtain an approximate solution of Eq. (14)

$$\alpha_1(s, s_0) = \lambda [1 - 2 I_{1,1}(s, s_0)]^{-1}. \quad (14)$$

The function $I_{1,1}(s, s_0)$ takes into account correctly the singularities corresponding to the fact that all propagators of triangle diagrams like those of Figs. 1d and 1e reduce to zero. The right-hand side of (14) may have a pole in s , which corresponds to a bound state of the three quarks. The choice of the cutoff Λ makes it possible to fix the value of the mass of the Δ isobar.

Baryons of S -wave multiplets have a completely symmetric spin-flavor part of the wave function, and spin $\frac{3}{2}$ corresponds to the decuplet which has a symmetric flavor part of the wave function. Octet states have spin $\frac{1}{2}$ and a mixed symmetry of the flavor function.

In analogy with the case of the Δ isobar we can obtain the rescattering amplitudes for all S -wave states with $J^P = \frac{3}{2}^+$, which include quarks of various flavors. These amplitudes will satisfy systems of integral equations. In considering the $J^P = \frac{1}{2}^+$ octet we must include the integration of the quarks in the 0^+ and 1^+ states (in the colour state $\bar{3}_c$). Including all possible rescattering of each pair of quarks and grouping the terms according to the final states of the particles, we obtain the amplitudes A_0 and A_1 , which satisfy the corresponding systems of integral equations. If we choose the approximation in which two-particle and triangle singularities are taken into account, and if all functions which depend on the physical region of the Dalitz plot, the problem of solving the system of integral equations reduces to one of solving simple algebraic equations.

In our calculation the quark masses $m_u = m_d = m$ and m_s are not uniquely determined. In order to fix m and m_s anyhow, we make the simple assumption that $m = \frac{1}{3}m_{\Delta}(1232)$ $m = \frac{1}{3}m_{\Omega}(1672)$. The strange quark breaks the flavor $SU(3)_f$ symmetry (6).

In Ref. [32] we consider two versions of calculations. In the first version the $SU(3)_f$ symmetry is broken by the scale shift of the dimensional parameters. A single cutoff parameter in pair energy is introduced for all diquark states $\lambda_1 = 12.2$.

In the Table I the calculated masses of the S -wave baryons are shown. In the first version we use only three parameters: the subenergy cutoff λ and the vertex function g_0, g_1 , which corresponds to the quark-quark interaction in 0^+ and 1^+ states. In this case the mass values of strange baryons with $J^P = \frac{1}{2}^+$ are less than the experimental ones. This means that the contribution color-magnetic is too large. In the second version we introduce four parameters: cutoff λ_0, λ_1 and the vertex function g_0, g_1 . We decrease the color-magnetic interaction in 0^+ strange channels and calculated mass values of two baryonic multiplets $J^P = \frac{1}{2}^+, \frac{3}{2}^+$ are in good agreement with the experimental data [40].

The essential difference between Σ and Λ is the spin of the lighter diquark. The model explains both the sign and magnitude of this mass splitting.

The suggested method of the approximate solution of the relativistic three-quark equations allows us to calculate the S -wave baryons mass spectrum. The interaction constants, determined the baryons spectrum in our model, are similar to ones in the bootstrap quark model of S -wave mesons [38]. The diquark interaction forces are defined by the gluon exchange. The relative contribution of the instanton-induced interaction is less than that with the gluon exchange. This is the consequence of $1/N_c$ -expansion [38].

The gluon exchange corresponds to the color-magnetic interaction, which is responsible for the spin-spin splitting in the hadron models. The sign of the color-magnetic term is such as to made any baryon of spin $\frac{3}{2}$ heavier than its spin- $\frac{1}{2}$ counterpart (containing the same flavors).

We manage with quarks as with real particles. However, in the soft region, the quark diagrams should be treated as spectral integrals over quark masses with the spectral density $\rho(m^2)$: the integration over quark masses in the amplitudes puts away the quark singularities and introduced the hadron ones. One can believe that the approximation:

$$\rho(m^2) \rightarrow \delta(m^2 - m_q^2) \quad (15)$$

could be possible for the low-lying hadrons (here m_q is the "mass" of the constituent quark).

We hope the approach given by (15) is sufficiently good for the calculation of the low-lying baryons being carried out here. The problem of distribution over quark masses is important when one considers that the high-excited states need spectral studies.

III. Six-quark amplitudes of the baryonia.

We derive the relativistic six-quark equations in the framework of the dispersion relation technique. We use only planar diagrams; the other diagrams due to the rules of $1/N_c$ expansion [41 – 43] are neglected. The current generates a six-quark system. The correct equations for the amplitude are obtained by taking into account all possible subamplitudes. It corresponds to the division of complete system into subsystems with a smaller number of particles. Then one should represent a six-particle amplitude as a sum of 15 subamplitudes:

$$A = \sum_{\substack{i < j \\ i, j = 1}}^6 A_{ij}. \quad (16)$$

This defines the division of the diagrams into groups according to the certain pair interaction of particles. The total amplitude can be represented graphically as a sum of diagrams. We need to consider only one group of diagrams and the amplitude corresponding to them, for example A_{12} . We shall consider the derivation of the relativistic generalization of the Faddeev-Yakubovsky approach.

In our case the low-lying baryonia are considered. We take into account the pairwise interaction of all quarks and antiquarks in the baryonia.

For instance, we consider the 1^{uu} -diquarks with spin-parity $J^P = 1^+$ for the baryonium content $uuu\bar{d}\bar{d}\bar{d}$ (Fig. 3). The set of diagrams associated with the amplitude A_{12} can further be broken down into five groups corresponding to subamplitudes: $A_1^{1^{uu}}(s, s_{12345}, s_{1234}, s_{123}, s_{12})$, $A_1^{1^{\bar{d}\bar{d}}}(s, s_{12345}, s_{1234}, s_{123}, s_{12})$, $A_1^{1^{u\bar{d}}}(s, s_{12345}, s_{1234}, s_{123}, s_{12})$, baryonium $A_2^{1^{uu}1^{\bar{d}\bar{d}}}(s, s_{12345}, s_{1234}, s_{12}, s_{34})$, $A_3^{1^{uu}1^{u\bar{d}}1^{\bar{d}\bar{d}}}(s, s_{12345}, s_{12}, s_{34}, s_{56})$. Here s_{ik} is the two-particle subenergy squared, s_{ijk} corresponds to the energy squared of particles i, j, k , s_{ijkl} is the energy squared of particles i, j, k, l , s_{ijklm} corresponds to the energy squared of particles i, j, k, l, m and s is the system total energy squared.

The amplitude $A_1^{1^{uu}}(s, s_{12345}, s_{1234}, s_{123}, s_{12})$ consists of the five color sub-structures: the diquark 1^{uu} in the color state $\bar{3}_c$, the quark u in the color state 3_c , the three \bar{d} antiquarks each in color

state $\bar{3}_c$, therefore we obtain $\bar{3}_c \times 3_c = 1_c + 8_c$, $\bar{3}_c \times \bar{3}_c \times \bar{3}_c = 1_c + 8_c + 8_c + 10_c^*$. Then we consider the total color singlet.

The baryonium amplitude $A_2^{1^{uu}1^{\bar{d}\bar{d}}}(s, s_{12345}, s_{1234}, s_{12}, s_{34})$ contains the diquark in the color state $\bar{3}_c$, antiquark $1^{\bar{d}\bar{d}}$ in the color state 3_c , u -quark in the color state 3_c , \bar{d} -antiquark in color state $\bar{3}_c$. We use the following equations: $\bar{3}_c \times 3_c = 1_c + 8_c$, $3_c \times \bar{3}_c = 1_c + 8_c$. Then the baryonium amplitude is the total singlet.

The amplitude $A_3^{1^{uu}1^{\bar{d}\bar{d}}1^{\bar{d}\bar{d}}}(s, s_{12345}, s_{12}, s_{34}, s_{56})$ consists of the diquark 1^{uu} in the color state $\bar{3}_c$, antiquark $1^{\bar{d}\bar{d}}$ in color state 3_c and noncolor state $1^{u\bar{d}}$, therefore the total color singlet can be constructed.

The subamplitudes $A_1^{1^{\bar{d}\bar{d}}}(s, s_{12345}, s_{1234}, s_{123}, s_{12})$ and $A_1^{1^{u\bar{d}}}(s, s_{12345}, s_{1234}, s_{123}, s_{12})$ are also the color singlets.

The system of graphical equations Fig. 3 is determined using the selfconsistent method. The coefficients are determined by the permutation of quarks [44, 45]. We should discuss the coefficient multiplying of the diagrams in the equations of Fig. 3. For example, we consider the first subamplitude $A_1^{1^{uu}}(s, s_{12345}, s_{1234}, s_{123}, s_{12})$. In the Fig. 3 the first coefficient is equal to 2 (permutation particles 1 and 2). The second coefficient is equal to $6 = 2$ (permutation particles 1 and 2) $\times 3$ (we consider the third, the fifth, the sixth particles). The similar approach allows us to take into account the coefficients in all equations.

In order to represent the subamplitudes $A_1^{1^{uu}}$, $A_1^{1^{u\bar{d}}}$, $A_1^{1^{\bar{d}\bar{d}}}$, $A_2^{1^{uu}1^{\bar{d}\bar{d}}}$, $A_3^{1^{uu}1^{\bar{d}\bar{d}}1^{\bar{d}\bar{d}}}$ in the form of dispersion relations, it is necessary to define the amplitudes of qq and $q\bar{q}$ interactions. This is similar to the three quark case (Sec. II).

We use the results of our relativistic quark model [38] and write down the pair quark amplitudes in the form:

$$a_n(s_{ik}) = \frac{G_n^2(s_{ik})}{1 - B_n(s_{ik})}, \quad (17)$$

$$B_n(s_{ik}) = \frac{\frac{(m_i+m_k)^2\Lambda}{4}}{(m_i+m_k)^2} \int \frac{ds'_{ik} \rho_n(s'_{ik}) G_n^2(s'_{ik})}{\pi (s'_{ik} - s_{ik})}. \quad (18)$$

Here $G_n(s_{ik})$ are the diquark vertex functions (Table V). The vertex functions are determined by the contribution of the crossing channels. The vertex functions satisfy the Fierz relations. These vertex functions are generated from g_V , $g_V^{(s)}$ and $g_V^{(ss)}$. $B_n(s_{ik})$ and $\rho_n(s_{ik})$ are the Chew-Mandelstam functions with cutoff Λ [34] and the phase spaces, respectively:

$$\begin{aligned} \rho_n(s_{ik}, J^{PC}) &= \left(\alpha(n, J^{PC}) \frac{s_{ik}}{(m_i + m_k)^2} + \beta(n, J^{PC}) + \delta(n, J^{PC}) \frac{(m_i - m_k)^2}{s_{ik}} \right) \\ &\times \frac{\sqrt{(s_{ik} - (m_i + m_k)^2)(s_{ik} - (m_i - m_k)^2)}}{s_{ik}}. \end{aligned} \quad (19)$$

The coefficients $\alpha(n, J^{PC})$, $\beta(n, J^{PC})$ and $\delta(n, J^{PC})$ are given in Table V.

Here $n = 1$ corresponds to $q\bar{q}$ -pairs with $J^P = 0^-$, $n = 2$ corresponds to the $q\bar{q}$ -pairs with $J^P = 1^-$, $n = 3$ defines the qq -pairs with $J^P = 0^+$, $n = 4$ corresponds to $J^P = 1^+$ qq states.

In the case in question the interacting quarks do not produce a bound state, therefore the integration in (20) – (24) is carried out from the threshold $(m_i + m_k)^2$ to the cutoff $\Lambda(ik)$.

The coupled integral equations correspond to Fig. 3 can be described as:

$$\begin{aligned}
A_1^{1uu}(s, s_{12345}, s_{1234}, s_{123}, s_{12}) &= \frac{\lambda_1 B_{1uu}(s_{12})}{[1 - B_{1uu}(s_{12})]} + 2\hat{J}_1(s_{12}, 1^{uu})A_1^{1uu}(s, s_{12345}, s_{1234}, s_{123}, s'_{13}) \\
&+ 6\hat{J}_1(s_{12}, 1^{uu})A_1^{1u\bar{d}}(s, s_{12345}, s_{1234}, s_{123}, s'_{13}), \tag{20}
\end{aligned}$$

$$\begin{aligned}
A_1^{1\bar{d}\bar{d}}(s, s_{12345}, s_{1234}, s_{123}, s_{12}) &= \frac{\lambda_1 B_{1\bar{d}\bar{d}}(s_{12})}{[1 - B_{1\bar{d}\bar{d}}(s_{12})]} + 2\hat{J}_1(s_{12}, 1^{\bar{d}\bar{d}})A_1^{1\bar{d}\bar{d}}(s, s_{12345}, s_{1234}, s_{123}, s'_{13}) \\
&+ 6\hat{J}_1(s_{12}, 1^{\bar{d}\bar{d}})A_1^{1u\bar{d}}(s, s_{12345}, s_{1234}, s_{123}, s'_{13}), \tag{21}
\end{aligned}$$

$$\begin{aligned}
A_1^{1u\bar{d}}(s, s_{12345}, s_{1234}, s_{123}, s_{12}) &= \frac{\lambda_1 B_{1u\bar{d}}(s_{12})}{[1 - B_{1u\bar{d}}(s_{12})]} + 2\hat{J}_1(s_{12}, 1^{u\bar{d}})A_1^{1u\bar{d}}(s, s_{12345}, s_{1234}, s_{123}, s'_{13}) \\
&+ 2\hat{J}_1(s_{12}, 1^{u\bar{d}})A_1^{1\bar{d}\bar{d}}(s, s_{12345}, s_{1234}, s_{123}, s'_{13}) \\
&+ 4\hat{J}_1(s_{12}, 1^{u\bar{d}})A_1^{1u\bar{d}}(s, s_{12345}, s_{1234}, s_{123}, s'_{13}) \\
&+ 4\hat{J}_2(s_{12}, 1^{u\bar{d}})A_2^{1uu1\bar{d}\bar{d}}(s, s_{12345}, s_{1234}, s'_{13}, s'_{24}), \tag{22}
\end{aligned}$$

$$\begin{aligned}
A_2^{1uu1\bar{d}\bar{d}}(s, s_{12345}, s_{1234}, s_{12}, s_{34}) &= \frac{\lambda_2 B_{1uu}(s_{12})B_{1\bar{d}\bar{d}}(s_{34})}{[1 - B_{1uu}(s_{12})][1 - B_{1\bar{d}\bar{d}}(s_{34})]} \\
&+ 2\hat{J}_4(s_{12}, s_{34}, 1^{uu}, 1^{\bar{d}\bar{d}})A_1^{1uu}(s, s_{12345}, s_{1235}, s_{125}, s'_{15}) \\
&+ 2\hat{J}_4(s_{12}, s_{34}, 1^{\bar{d}\bar{d}}, 1^{uu})A_1^{1\bar{d}\bar{d}}(s, s_{12345}, s_{1235}, s_{125}, s'_{15}) \\
&+ 4\hat{J}_3(s_{12}, s_{34}, 1^{uu}, 1^{\bar{d}\bar{d}})A_1^{1u\bar{d}}(s, s_{12345}, s_{1234}, s'_{123}, s'_{23}) \\
&+ 4\hat{J}_6(s_{12}, s_{34}, 1^{uu}, 1^{uu})A_2^{1uu1\bar{d}\bar{d}}(s, s_{12456}, s_{1456}, s'_{15}, s'_{46}) \\
&+ 4\hat{J}_8(s_{12}, s_{34}, 1^{uu}, 1^{\bar{d}\bar{d}})A_3^{1uu1u\bar{d}1\bar{d}\bar{d}}(s, s_{12345}, s'_{15}, s'_{23}, s'_{46}), \tag{23}
\end{aligned}$$

$$\begin{aligned}
A_3^{1uu1u\bar{d}1\bar{d}\bar{d}}(s, s_{12345}, s_{12}, s_{34}, s_{56}) &= \frac{\lambda_3 B_{1uu}(s_{12})B_{1u\bar{d}}(s_{34})B_{1\bar{d}\bar{d}}(s_{56})}{[1 - B_{1uu}(s_{12})][1 - B_{1u\bar{d}}(s_{34})][1 - B_{1\bar{d}\bar{d}}(s_{56})]} \\
&+ 2\hat{J}_9(s_{12}, s_{34}, s_{56}, 1^{uu}, 1^{u\bar{d}}, 1^{\bar{d}\bar{d}})A_1^{1uu}(s, s_{12345}, s_{1234}, s'_{123}, s'_{23}) \\
&+ 2\hat{J}_9(s_{12}, s_{34}, s_{56}, 1^{\bar{d}\bar{d}}, 1^{u\bar{d}}, 1^{uu})A_1^{1\bar{d}\bar{d}}(s, s_{12345}, s_{1234}, s'_{123}, s'_{23}) \\
&+ 2\hat{J}_9(s_{12}, s_{34}, s_{56}, 1^{uu}, 1^{u\bar{d}}, 1^{\bar{d}\bar{d}})A_1^{1u\bar{d}}(s, s_{12345}, s_{1234}, s'_{123}, s'_{23}) \\
&+ 4\hat{J}_9(s_{12}, s_{34}, s_{56}, 1^{uu}, 1^{\bar{d}\bar{d}}, 1^{u\bar{d}})A_1^{1u\bar{d}}(s, s_{12345}, s_{1234}, s'_{123}, s'_{23})
\end{aligned}$$

$$\begin{aligned}
& + 2\hat{J}_9(s_{12}, s_{34}, s_{56}, 1^{\bar{d}\bar{d}}, 1^{u\bar{d}}, 1^{uu})A_1^{1^{u\bar{d}}}(s, s_{12345}, s_{1234}, s'_{123}, s'_{23}) \\
& + 4\hat{J}_{10}(s_{12}, s_{34}, s_{56}, 1^{uu}, 1^{u\bar{d}}, 1^{\bar{d}\bar{d}})A_2^{1^{uu}1^{\bar{d}\bar{d}}}(s, s_{12345}, s_{2345}, s'_{23}, s'_{45}),
\end{aligned} \tag{24}$$

where

$$\hat{J}_1(s_{12}, i) = \frac{G_i(s_{12})}{[1 - B_i(s_{12})]} \int_{(m_1+m_2)^2}^4 \frac{ds'_{12}}{\pi} \frac{G_i(s'_{12})\rho_i(s'_{12})}{s'_{12} - s_{12}} \int_{-1}^{+1} \frac{dz_1(1)}{2}, \tag{25}$$

$$\begin{aligned}
\hat{J}_2(s_{12}, i) &= \frac{G_i(s_{12})}{[1 - B_i(s_{12})]} \int_{(m_1+m_2)^2}^4 \frac{ds'_{12}}{\pi} \frac{G_i(s'_{12})\rho_i(s'_{12})}{s'_{12} - s_{12}} \frac{1}{2\pi} \int_{-1}^{+1} \frac{dz_1(2)}{2} \int_{-1}^{+1} \frac{dz_2(2)}{2} \\
&\times \int_{z_3(2)^-}^{z_3(2)^+} dz_3(2) \frac{1}{\sqrt{1 - z_1^2(2) - z_2^2(2) - z_3^2(2) + 2z_1(2)z_2(2)z_3(2)}},
\end{aligned} \tag{26}$$

$$\begin{aligned}
\hat{J}_3(s_{12}, s_{34}, i, j) &= \frac{G_i(s_{12})G_j(s_{34})}{[1 - B_i(s_{12})][1 - B_j(s_{34})]} \int_{(m_1+m_2)^2}^4 \frac{ds'_{12}}{\pi} \frac{G_i(s'_{12})\rho_i(s'_{12})}{s'_{12} - s_{12}} \\
&\times \int_{(m_3+m_4)^2}^4 \frac{ds'_{34}}{\pi} \frac{G_j(s'_{34})\rho_j(s'_{34})}{s'_{34} - s_{34}} \int_{-1}^{+1} \frac{dz_1(3)}{2} \int_{-1}^{+1} \frac{dz_2(3)}{2},
\end{aligned} \tag{27}$$

$$\hat{J}_4(s_{12}, s_{34}, i, j) = \frac{B_j(s_{34})}{[1 - B_j(s_{34})]} \hat{J}_1(s_{12}, i), \tag{28}$$

$$\hat{J}_6(s_{12}, s_{34}, i, j) = \hat{J}_1(s_{12}, i) \cdot \hat{J}_1(s_{34}, j), \tag{29}$$

$$\begin{aligned}
\hat{J}_8(s_{12}, s_{34}, i, j) &= \frac{G_i(s_{12})G_j(s_{34})}{[1 - B_i(s_{12})][1 - B_j(s_{34})]} \int_{(m_1+m_2)^2}^4 \frac{ds'_{12}}{\pi} \frac{G_i(s'_{12})\rho_i(s'_{12})}{s'_{12} - s_{12}} \\
&\times \int_{(m_3+m_4)^2}^4 \frac{ds'_{34}}{\pi} \frac{G_j(s'_{34})\rho_j(s'_{34})}{s'_{34} - s_{34}} \\
&\times \frac{1}{(2\pi)^2} \int_{-1}^{+1} \frac{dz_1(8)}{2} \int_{-1}^{+1} \frac{dz_2(8)}{2} \int_{-1}^{+1} \frac{dz_3(8)}{2} \int_{z_4(8)^-}^{z_4(8)^+} dz_4(8) \int_{-1}^{+1} \frac{dz_5(8)}{2} \int_{z_6(8)^-}^{z_6(8)^+} dz_6(8) \\
&\times \frac{1}{\sqrt{1 - z_1^2(8) - z_3^2(8) - z_4^2(8) + 2z_1(8)z_3(8)z_4(8)}} \\
&\times \frac{1}{\sqrt{1 - z_2^2(8) - z_5^2(8) - z_6^2(8) + 2z_2(8)z_5(8)z_6(8)}},
\end{aligned} \tag{30}$$

$$\hat{J}_9(s_{12}, s_{34}, s_{56}, i, j, k) = \frac{B_k(s_{56})}{[1 - B_k(s_{56})]} \hat{J}_3(s_{12}, s_{34}, i, j), \quad (31)$$

$$\begin{aligned} \hat{J}_{10}(s_{12}, s_{34}, s_{56}, i, j, k) &= \frac{G_i(s_{12})G_j(s_{34})G_k(s_{56})}{[1 - B_i(s_{12})][1 - B_j(s_{34})][1 - B_k(s_{56})]} \int_{(m_1+m_2)^2}^{\frac{(m_1+m_2)^2 \Lambda_i}{4}} \frac{ds'_{12}}{\pi} \frac{G_i(s'_{12})\rho_i(s'_{12})}{s'_{12} - s_{12}} \\ &\times \int_{(m_3+m_4)^2}^{\frac{(m_3+m_4)^2 \Lambda_j}{4}} \frac{ds'_{34}}{\pi} \frac{G_j(s'_{34})\rho_j(s'_{34})}{s'_{34} - s_{34}} \int_{(m_5+m_6)^2}^{\frac{(m_5+m_6)^2 \Lambda_k}{4}} \frac{ds'_{56}}{\pi} \frac{G_k(s'_{56})\rho_k(s'_{56})}{s'_{56} - s_{56}} \\ &\times \frac{1}{2\pi} \int_{-1}^{+1} \frac{dz_1(10)}{2} \int_{-1}^{+1} \frac{dz_2(10)}{2} \int_{-1}^{+1} \frac{dz_3(10)}{2} \int_{-1}^{+1} \frac{dz_4(10)}{2} \int_{z_5(1-)^-}^{z_5(10)^+} dz_5(10) \\ &\times \frac{1}{\sqrt{1 - z_1^2(10) - z_4^2(10) - z_5^2(10) + 2z_1(10)z_4(10)z_5(10)}}. \end{aligned} \quad (32)$$

In the equation (25) $z_1(1)$ is the cosine of the angle between the relative momentum of particles 1 and 2 in the intermediate state and the momentum of the particle 3 in the final state taken in the c.m. of particles 1 and 2. We can go from the integration of the cosine of the angle $dz_1(1)$ to the integration over the subenergy ds'_{13} .

In Eq. (26) $z_1(2)$ is the cosine of the angle between the relative momentum of particles 1 and 2 in the intermediate state and the momentum of the particle 3 in the final state taken in the c.m. of particles 1 and 2, $z_2(2)$ is the cosine of the angle between the momenta of particles 3 and 4 in the final state of c.m. of particles 1 and 2, $z_3(2)$ is cosine of the angle between the relative momentum of particles 1 and 2 in the intermediate state and the momentum of the particle 4 in the final state of c.m. of particles 1 and 2. Then we pass from $dz_1(2)dz_2(2)dz_3(2)$ to $ds'_{13}ds'_{34}ds'_{24}$.

In Eq. (27) $z_1(3)$ is the cosine of the angle between the relative momentum of particles 1, 2 in the intermediate state and the relative momentum of particles 3, 4 in the intermediate state in c.m. of particles 3 and 4; $z_2(3)$ is the cosine of the angle between momentum of particle 3 in the intermediate state and relative momentum of particles 1, 2 in the intermediate state in c.m. 1 and 2. We pass from $dz_1(3)dz_2(3)$ to $ds'_{123}ds'_{23}$. The similar method is used for the functions (28), (29), (31).

In Eq. (30) $z_1(8)$ is the cosine of the angle between momentum of particle 5 in the final state and the relative momentum of particles 1, 2 in the intermediate state in c.m. of particles 1 and 2; $z_2(8)$ is the cosine of the angle between the relative momentum of particles 1, 2 in the intermediate state and the relative momentum of particles 3, 4 in the intermediate state in c.m. of particles 3 and 4; $z_3(8)$ is the cosine of the angle between momentum of particle 3 in the intermediate state and the momentum of particle 5 in the final state in c.m. of particles 1 and 2; $z_4(8)$ is the cosine of the angle between the momentum of particle 3 in the intermediate state and the relative momentum of particles 1, 2 in the intermediate state in c.m. of particles 1 and 2; $z_5(8)$ is the cosine of angle between momentum of particle 6 in the final state and the relative momentum of particles 1, 2 in the intermediate state in c.m. of particles 3 and 4; $z_6(8)$ is the cosine of the angle between momentum of particle 6 in the final state and the relative momentum of particles 3, 4 in the intermediate state in c.m. of particles 3 and 4. We pass from $dz_1(8)dz_2(8)dz_3(8)dz_4(8)dz_5(8)dz_6(8)$ to $ds'_{15}ds'_{123}ds'_{35}ds'_{23}ds'_{126}ds'_{46}$.

In Eq. (32) $z_1(10)$ is the cosine of angle between relative momentum of particles 1, 2 in the intermediate state and the relative momentum of particles 3, 4 in the intermediate state in c.m.

of particles 3 and 4; $z_2(10)$ is the cosine of angle between the relative momentum of particles 1, 2 in the intermediate state and momentum of particle 3 in the final state in c.m. of particles 1 and 2; $z_3(10)$ is the cosine of the angle between the relative momentum of the particles 3, 4 in the intermediate state and the relative momentum of particles 5, 6 in the intermediate state in c.m. of particles 5 and 6; $z_4(10)$ is the cosine of angle between relative momentum of particles 1, 2 in the intermediate state and the momentum of particle 5 in the final state in c.m. of particles 3 and 4; $z_5(10)$ is the cosine of the angle between the relative momentum of the particles 3, 4 in the intermediate state and the momentum of particle 5 in the final state in c.m. of particles 3 and 4. We pass from $dz_1(10)dz_2(10)dz_3(10)dz_4(10)dz_5(10)$ to $ds'_{123}ds'_{23}ds'_{345}ds'_{125}ds'_{45}$.

Let us extract two- and three-particle singularities in the amplitudes $A_1^{1uu}(s, s_{12345}, s_{1234}, s_{123}, s_{12})$, $A_1^{1\bar{d}\bar{d}}(s, s_{12345}, s_{1234}, s_{123}, s_{12})$, $A_1^{1u\bar{d}}(s, s_{12345}, s_{1234}, s_{123}, s_{12})$, $A_2^{1uu1\bar{d}\bar{d}}(s, s_{12345}, s_{1234}, s_{12}, s_{34})$, $A_3^{1uu1u\bar{d}1\bar{d}\bar{d}}(s, s_{12345}, s_{12}, s_{34}, s_{56})$:

$$A_1^{1uu}(s, s_{12345}, s_{1234}, s_{123}, s_{12}) = \frac{\alpha_1^{1uu}(s, s_{12345}, s_{1234}, s_{123}, s_{12})B_{1uu}(s_{12})}{[1 - B_{1uu}(s_{12})]}, \quad (33)$$

$$A_1^{1\bar{d}\bar{d}}(s, s_{12345}, s_{1234}, s_{123}, s_{12}) = \frac{\alpha_1^{1\bar{d}\bar{d}}(s, s_{12345}, s_{1234}, s_{123}, s_{12})B_{1\bar{d}\bar{d}}(s_{12})}{[1 - B_{1\bar{d}\bar{d}}(s_{12})]}, \quad (34)$$

$$A_1^{1u\bar{d}}(s, s_{12345}, s_{1234}, s_{123}, s_{12}) = \frac{\alpha_1^{1u\bar{d}}(s, s_{12345}, s_{1234}, s_{123}, s_{12})B_{1u\bar{d}}(s_{12})}{[1 - B_{1u\bar{d}}(s_{12})]}, \quad (35)$$

$$A_2^{1uu1\bar{d}\bar{d}}(s, s_{12345}, s_{1234}, s_{12}, s_{34}) = \frac{\alpha_2^{1uu1\bar{d}\bar{d}}(s, s_{12345}, s_{1234}, s_{12}, s_{34})B_{1uu}(s_{12})B_{1\bar{d}\bar{d}}(s_{34})}{[1 - B_{1uu}(s_{12})][1 - B_{1\bar{d}\bar{d}}(s_{34})]}, \quad (36)$$

$$A_3^{1uu1u\bar{d}1\bar{d}\bar{d}}(s, s_{12345}, s_{12}, s_{34}, s_{56}) = \frac{\alpha_3^{1uu1u\bar{d}1\bar{d}\bar{d}}(s, s_{12345}, s_{12}, s_{34}, s_{56})B_{1uu}(s_{12})B_{1u\bar{d}}(s_{34})B_{1\bar{d}\bar{d}}(s_{56})}{[1 - B_{1uu}(s_{12})][1 - B_{1u\bar{d}}(s_{34})][1 - B_{1\bar{d}\bar{d}}(s_{56})]}. \quad (37)$$

We do not extract four-particles singularities, because they are weaker than two- and three-particle singularities.

We used the classification of singularities, which was proposed in paper [39]. The construction of the approximate solution of Eqs. (33) – (37) is based on the extraction of the leading singularities of the amplitudes. The main singularities in $s_{ik} = (m_i + m_k)^2$ are from pair rescattering of the particles i and k . First of all there are threshold square-root singularities. Also possible are pole singularities which correspond to the bound states. The diagrams of Fig. 3 apart from two-particle singularities have triangular singularities and the singularities defining the interactions of four, five and six particles. Such classification allows us to search the corresponding solution of Eqs. (20) – (24) by taking into account some definite number of leading singularities and neglecting all the weaker ones. We consider the approximation which defines two-particle, triangle and four-, five- and six-particle singularities. The contribution of two-particle and triangle singularities are more important, but we must take into account also the other singularities.

The five functions α_i are the smooth functions of s_{ik} , s_{ijk} , s_{ijkl} , s_{ijklm} as compared with the singular part of the amplitudes, hence they can be expanded in a series in the singularity point and only the first term of this series should be employed further. Using this classification, one defines the reduced amplitudes α_i as well as the B -functions in the middle point of physical region of Dalitz-plot at the point s_0 :

$$s_0 = \frac{s + 4 \sum_{i=1}^6 m_i^2}{\sum_{\substack{i,k=1 \\ i < k}}^6 m_{ik}^2}, \quad (38)$$

$$s_{123} = s_0 \sum_{\substack{i,k=1 \\ i < k}}^3 m_{ik}^2 - \sum_{i=1}^3 m_i^2, \quad (39)$$

$$s_{1234} = s_0 \sum_{\substack{i,k=1 \\ i < k}}^4 m_{ik}^2 - 2 \sum_{i=1}^4 m_i^2. \quad (40)$$

Such choice of point s_0 allows us to replace integral equations (20) – (24) (Fig. 3) by the algebraic equations (41) – (45), respectively:

$$\alpha_1^{1^{uu}} = \lambda + 2I_1(1^{uu}1^{uu})\alpha_1^{1^{uu}} + 6I_1(1^{uu}1^{u\bar{d}})\alpha_1^{1^{u\bar{d}}}, \quad (41)$$

$$\alpha_1^{1^{\bar{d}\bar{d}}} = \lambda + 2I_1(1^{\bar{d}\bar{d}}1^{\bar{d}\bar{d}})\alpha_1^{1^{\bar{d}\bar{d}}} + 6I_1(1^{\bar{d}\bar{d}}1^{u\bar{d}})\alpha_1^{1^{u\bar{d}}}, \quad (42)$$

$$\alpha_1^{1^{u\bar{d}}} = \lambda + 2I_1(1^{u\bar{d}}1^{uu})\alpha_1^{1^{uu}} + 2I_1(1^{u\bar{d}}1^{\bar{d}\bar{d}})\alpha_1^{1^{\bar{d}\bar{d}}} + 4I_1(1^{u\bar{d}}1^{u\bar{d}})\alpha_1^{1^{u\bar{d}}} + 4I_2(1^{u\bar{d}}1^{uu}1^{\bar{d}\bar{d}})\alpha_2^{1^{uu}1^{\bar{d}\bar{d}}}, \quad (43)$$

$$\begin{aligned} \alpha_2^{1^{uu}1^{\bar{d}\bar{d}}} &= \lambda + 2I_4(1^{uu}1^{\bar{d}\bar{d}}1^{uu})\alpha_1^{1^{uu}} + 2I_4(1^{\bar{d}\bar{d}}1^{uu}1^{\bar{d}\bar{d}})\alpha_1^{1^{\bar{d}\bar{d}}} + 4I_3(1^{uu}1^{\bar{d}\bar{d}}1^{u\bar{d}})\alpha_1^{1^{u\bar{d}}} \\ &+ 4I_6(1^{uu}1^{\bar{d}\bar{d}}1^{uu}1^{\bar{d}\bar{d}})\alpha_2^{1^{uu}1^{\bar{d}\bar{d}}} + 4I_8(1^{uu}1^{\bar{d}\bar{d}}1^{uu}1^{u\bar{d}}1^{\bar{d}\bar{d}})\alpha_3^{1^{uu}1^{u\bar{d}}1^{\bar{d}\bar{d}}}, \end{aligned} \quad (44)$$

$$\begin{aligned} \alpha_3^{1^{uu}1^{u\bar{d}}1^{\bar{d}\bar{d}}} &= \lambda + 2I_9(1^{uu}1^{u\bar{d}}1^{\bar{d}\bar{d}}1^{uu})\alpha_1^{1^{uu}} + 2I_9(1^{\bar{d}\bar{d}}1^{u\bar{d}}1^{uu}1^{\bar{d}\bar{d}})\alpha_1^{1^{\bar{d}\bar{d}}} + 2I_9(1^{uu}1^{u\bar{d}}1^{\bar{d}\bar{d}}1^{u\bar{d}})\alpha_1^{1^{u\bar{d}}} \\ &+ 4I_9(1^{uu}1^{\bar{d}\bar{d}}1^{u\bar{d}}1^{u\bar{d}})\alpha_1^{1^{u\bar{d}}} + 2I_9(1^{\bar{d}\bar{d}}1^{u\bar{d}}1^{uu}1^{u\bar{d}})\alpha_1^{1^{u\bar{d}}} + 4I_{10}(1^{uu}1^{u\bar{d}}1^{\bar{d}\bar{d}}1^{uu}1^{\bar{d}\bar{d}})\alpha_2^{1^{uu}1^{\bar{d}\bar{d}}}, \end{aligned} \quad (45)$$

where λ_i are the current constants. We used the functions $I_1, I_2, I_3, I_4, I_6, I_8, I_9, I_{10}$:

$$\begin{aligned} I_1(ij) &= \frac{B_j(s_0^{13})}{B_i(s_0^{12})} \frac{\int_0^4 \frac{ds'_{12}}{(m_1+m_2)^2} \frac{G_i^2(s_0^{12})\rho_i(s'_{12})}{\pi} \frac{1}{s'_{12} - s_0^{12}} \int_{-1}^{+1} \frac{dz_1(1)}{2} \frac{1}{1 - B_j(s'_{13})}}, \quad (46) \\ I_2(ijk) &= \frac{B_j(s_0^{13})B_k(s_0^{24})}{B_i(s_0^{12})} \frac{\int_0^4 \frac{ds'_{12}}{(m_1+m_2)^2} \frac{G_i^2(s_0^{12})\rho_i(s'_{12})}{\pi} \frac{1}{s'_{12} - s_0^{12}} \frac{1}{2\pi} \int_{-1}^{+1} \frac{dz_1(2)}{2} \int_{-1}^{+1} \frac{dz_2(2)}{2}} \\ &\times \int_{z_3(2)^-}^{z_3(2)^+} dz_3(2) \frac{1}{\sqrt{1 - z_1^2(2) - z_2^2(2) - z_3^2(2) + 2z_1(2)z_2(2)z_3(2)}} \end{aligned}$$

$$\times \frac{1}{1 - B_j(s'_{13})} \frac{1}{1 - B_k(s'_{24})}, \quad (47)$$

$$\begin{aligned} I_3(ijk) &= \frac{B_k(s_0^{23})}{B_i(s_0^{12})B_j(s_0^{34})} \int_{(m_1+m_2)^2}^4 \frac{ds'_{12}}{\pi} \frac{G_i^2(s_0^{12})\rho_i(s'_{12})}{s'_{12} - s_0^{12}} \\ &\times \int_{(m_3+m_4)^2}^4 \frac{ds'_{34}}{\pi} \frac{G_j^2(s_0^{34})\rho_j(s'_{34})}{s'_{34} - s_0^{34}} \int_{-1}^{+1} \frac{dz_1(3)}{2} \int_{-1}^{+1} \frac{dz_2(3)}{2} \frac{1}{1 - B_k(s'_{23})}, \end{aligned} \quad (48)$$

$$I_4(ijk) = I_1(ik), \quad (49)$$

$$I_6(ijkl) = I_1(ik) \cdot I_1(jl), \quad (50)$$

$$\begin{aligned} I_8(ijklm) &= \frac{B_k(s_0^{15})B_l(s_0^{23})B_m(s_0^{46})}{B_i(s_0^{12})B_j(s_0^{34})} \int_{(m_1+m_2)^2}^4 \frac{ds'_{12}}{\pi} \frac{G_i^2(s_0^{12})\rho_i(s'_{12})}{s'_{12} - s_0^{12}} \\ &\times \int_{(m_3+m_4)^2}^4 \frac{ds'_{34}}{\pi} \frac{G_j^2(s_0^{34})\rho_j(s'_{34})}{s'_{34} - s_0^{34}} \\ &\times \frac{1}{(2\pi)^2} \int_{-1}^{+1} \frac{dz_1(8)}{2} \int_{-1}^{+1} \frac{dz_2(8)}{2} \int_{-1}^{+1} \frac{dz_3(8)}{2} \int_{z_4(8)^-}^{z_4(8)^+} dz_4(8) \int_{-1}^{+1} \frac{dz_5(8)}{2} \int_{z_6(8)^-}^{z_6(8)^+} dz_6(8) \\ &\times \frac{1}{\sqrt{1 - z_1^2(8) - z_3^2(8) - z_4^2(8) + 2z_1(8)z_3(8)z_4(8)}} \\ &\times \frac{1}{\sqrt{1 - z_2^2(8) - z_5^2(8) - z_6^2(8) + 2z_2(8)z_5(8)z_6(8)}} \\ &\times \frac{1}{1 - B_k(s'_{15})} \frac{1}{1 - B_l(s'_{23})} \frac{1}{1 - B_m(s'_{46})}, \end{aligned} \quad (51)$$

$$I_9(ijkl) = I_3(ijl), \quad (52)$$

$$\begin{aligned} I_{10}(ijklm) &= \frac{B_l(s_0^{23})B_m(s_0^{45})}{B_i(s_0^{12})B_j(s_0^{34})B_k(s_0^{56})} \int_{(m_1+m_2)^2}^4 \frac{ds'_{12}}{\pi} \frac{G_i^2(s_0^{12})\rho_i(s'_{12})}{s'_{12} - s_0^{12}} \\ &\times \int_{(m_3+m_4)^2}^4 \frac{ds'_{34}}{\pi} \frac{G_j^2(s_0^{34})\rho_j(s'_{34})}{s'_{34} - s_0^{34}} \int_{(m_5+m_6)^2}^4 \frac{ds'_{56}}{\pi} \frac{G_k^2(s_0^{56})\rho_k(s'_{56})}{s'_{56} - s_0^{56}} \end{aligned}$$

$$\begin{aligned}
& \times \frac{1}{2\pi} \int_{-1}^{+1} \frac{dz_1(10)}{2} \int_{-1}^{+1} \frac{dz_2(10)}{2} \int_{-1}^{+1} \frac{dz_3(10)}{2} \int_{-1}^{+1} \frac{dz_4(10)}{2} \int_{z_5(1-)^-}^{z_5(10)^+} dz_5(10) \\
& \times \frac{1}{\sqrt{1 - z_1^2(10) - z_4^2(10) - z_5^2(10) + 2z_1(10)z_4(10)z_5(10)}} \\
& \times \frac{1}{1 - B_l(s'_{23})} \frac{1}{1 - B_m(s'_{45})}, \tag{53}
\end{aligned}$$

where i, j, k, l, m correspond to the diquarks with the spin-parity $J^P = 0^+, 1^+$ and mesons with the spin-parity $J^P = 0^-, 1^-$.

The other choices of point s_0 do not change essentially the contributions of α_i , therefore we omit the indices s_0^{ik} . Since the vertex functions depend only slightly on energy, it is possible to treat them as constants in our approximation.

The solutions of the system of equations are considered as:

$$\alpha_i(s) = F_i(s, \lambda_i)/D(s), \tag{54}$$

where zeros of $D(s)$ determinants define the masses of bound states of baryonia.

As example, we consider the equations for the quark content $uuu\bar{d}\bar{d}\bar{d}$ with the isospin $I = 3$ and the spin-parity $J^P = 3^-$ (Fig. 3). The similar equations have been calculated for the isospin $I = 0, 1, 2, 3$ and the spin-parity $J^P = 0^-, 1^-, 2^-, 3^-$. We take into account the u and d quarks.

In Appendix I the reduced amplitudes of baryonium $uud\bar{u}\bar{u}\bar{d}$ $IJ = 00$ are given.

IV. Calculation results.

The poles of the five reduced amplitudes α_i correspond to the bound state and determine the mass of the baryonium with the quark content $uuu\bar{d}\bar{d}\bar{d}$, with the isospin $I = 3$ and the spin-parity $J^P = 3^-$. The quark mass of model $m = 410 \text{ MeV}$ coincides with the ordinary baryon one in our model (Sec. II).

The model in question has only two parameters: the cutoff parameter $\Lambda = 11$ (similar to the three quark model (Sec. II)) and the gluon coupling constant $g = 0.314$. This parameter is determined by the baryonium mass $M = 1835 \text{ MeV}$. The estimation of theoretical error on the baryonia masses is 1 MeV . This result was obtained by the choice of model parameters.

We predict the degeneracy of the some states. In the Table II the calculated masses of non-strange baryonia are shown. The contributions of subamplitudes to the six-quark amplitude are shown in the Appendix I (for example, the baryonium with the mass $M = 1835 \text{ MeV}$). The states $(\Delta\bar{\Delta} + \Delta\bar{n} + n\bar{\Delta} + n\bar{n})$ and $(\Delta\bar{\Delta} + \Delta\bar{p} + p\bar{\Delta} + p\bar{p})$ with the isospin $I = 0$ and the spin-parity $J^P = 0^-$ possess the mass $M = 1835 \text{ MeV}$. For the $(\Delta\bar{\Delta} + n\bar{\Delta})$ and $(\Delta\bar{\Delta} + \Delta\bar{n})$ with the isospin $I = 1$ and the spin-parity $J^P = 0^-$ we obtained the mass $M = 1928 \text{ MeV}$.

We predict the degeneracy of baryonia $M(uud\bar{d}\bar{d}\bar{d}, I = 2) = M(uuu\bar{d}\bar{d}\bar{d}, I = 2) = M(udd\bar{d}\bar{d}\bar{d}, I = 1) = M(uuu\bar{u}\bar{u}\bar{d}, I = 1)$. For the states $M(uud\bar{u}\bar{d}\bar{d}, I = 1) = M(udd\bar{u}\bar{d}\bar{d}, I = 0) = M(uud\bar{u}\bar{u}\bar{d}, I = 0)$ and $M(uuu\bar{u}\bar{u}\bar{u}, I = 0) = M(ddd\bar{d}\bar{d}\bar{d}, I = 0)$ the degeneracy is also obtained.

V. Conclusion.

A somewhat simple picture of baryonium is that of a deuteron-like $N\bar{N}$ bound state or resonance, benefiting from the attractive potential mediated by the exchange of gluon [33]. We do not consider the influence of annihilation on the spectrum.

Entem and Fernandez, describing scattering data and mass shifts of $p\bar{p}$ levels in a constituent quark model, assign the threshold enhancement to final-state interaction [46, 47]. Zou and Chi-

and find that final state interaction makes an important contribution to the $p\bar{p}$ near threshold enhancement [48].

The baryonium state with $M = 1835 \text{ MeV}$ is considered as $p\bar{p}$ state [3] or the second radial excitation of η' meson [49].

In our case this state have following content $\Delta\bar{\Delta} + \Delta\bar{p} + p\bar{\Delta} + p\bar{p}$ with isospin $I = 0$ and spin-parity $J^P = 0^-$.

We calculated the masses of baryonia with the isospin $I = 0, 1, 2, 3$ and spin-parity $J^P = 0^-, 1^-, 2^-, 3^-$ (Table II).

The interesting reseach is the contribution of baryonia consisting of u, d, s -quarks and anti-quarks.

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Appendix I. The reduced amplitudes of baryonium $IJ = 00(1835)$.

$$\begin{aligned}
\alpha_1^{1^{uu}} &= \lambda + 2\alpha_1^{1^{u\bar{u}}} I_1(1^{uu}1^{u\bar{u}}) + 2\alpha_1^{1^{u\bar{d}}} I_1(1^{uu}1^{u\bar{d}}) + 2\alpha_1^{0^{ud}} I_1(1^{uu}0^{ud}) + 4\alpha_1^{0^{u\bar{u}}} I_1(1^{uu}0^{u\bar{u}}) \\
&+ 2\alpha_1^{0^{ud}} I_1(1^{uu}0^{u\bar{d}}) \\
\alpha_1^{1^{\bar{u}\bar{u}}} &= \lambda + 2\alpha_1^{1^{u\bar{u}}} I_1(1^{\bar{u}\bar{u}}1^{u\bar{u}}) + 2\alpha_1^{1^{d\bar{u}}} I_1(1^{\bar{u}\bar{u}}1^{d\bar{u}}) + 2\alpha_1^{0^{\bar{u}\bar{d}}} I_1(1^{\bar{u}\bar{u}}0^{\bar{u}\bar{d}}) + 4\alpha_1^{0^{u\bar{u}}} I_1(1^{\bar{u}\bar{u}}0^{u\bar{u}}) \\
&+ 2\alpha_1^{0^{d\bar{u}}} I_1(1^{\bar{u}\bar{u}}0^{d\bar{u}}) \\
\alpha_1^{1^{u\bar{u}}} &= \lambda + \alpha_1^{1^{uu}} I_1(1^{u\bar{u}}1^{uu}) + \alpha_1^{1^{\bar{u}\bar{u}}} I_1(1^{u\bar{u}}1^{\bar{u}\bar{u}}) + 2\alpha_1^{1^{u\bar{u}}} I_1(1^{u\bar{u}}1^{u\bar{u}}) + \alpha_1^{1^{u\bar{d}}} I_1(1^{u\bar{u}}1^{u\bar{d}}) + \alpha_1^{1^{d\bar{u}}} I_1(1^{u\bar{u}}1^{d\bar{u}}) \\
&+ \alpha_1^{0^{ud}} I_1(1^{u\bar{u}}0^{ud}) + \alpha_1^{0^{\bar{u}\bar{d}}} I_1(1^{u\bar{u}}0^{\bar{u}\bar{d}}) + 2\alpha_1^{0^{u\bar{u}}} I_1(1^{u\bar{u}}0^{u\bar{u}}) + \alpha_1^{0^{u\bar{d}}} I_1(1^{u\bar{u}}0^{u\bar{d}}) + \alpha_1^{0^{d\bar{u}}} I_1(1^{u\bar{u}}0^{d\bar{u}}) \\
&+ \alpha_2^{1^{uu}0^{\bar{u}\bar{d}}} I_2(1^{u\bar{u}}1^{uu}0^{\bar{u}\bar{d}}) + \alpha_2^{1^{\bar{u}\bar{u}}0^{ud}} I_2(1^{u\bar{u}}1^{\bar{u}\bar{u}}0^{ud}) + \alpha_2^{0^{ud}0^{\bar{u}\bar{d}}} I_2(1^{u\bar{u}}0^{ud}0^{\bar{u}\bar{d}}) \\
\alpha_1^{1^{u\bar{d}}} &= \lambda + \alpha_1^{1^{uu}} I_1(1^{u\bar{d}}1^{uu}) + \alpha_1^{1^{u\bar{u}}} I_1(1^{u\bar{d}}1^{u\bar{u}}) + \alpha_1^{1^{u\bar{d}}} I_1(1^{u\bar{d}}1^{u\bar{d}}) + \alpha_1^{1^{d\bar{d}}} I_1(1^{u\bar{d}}1^{d\bar{d}}) + \alpha_1^{0^{ud}} I_1(1^{u\bar{d}}0^{ud}) \\
&+ 2\alpha_1^{0^{\bar{u}\bar{d}}} I_1(1^{u\bar{d}}0^{\bar{u}\bar{d}}) + 2\alpha_1^{0^{u\bar{u}}} I_1(1^{u\bar{d}}0^{u\bar{u}}) + \alpha_1^{0^{u\bar{d}}} I_1(1^{u\bar{d}}0^{u\bar{d}}) + \alpha_1^{0^{d\bar{d}}} I_1(1^{u\bar{d}}0^{d\bar{d}}) \\
&+ 2\alpha_2^{1^{uu}0^{\bar{u}\bar{d}}} I_2(1^{u\bar{d}}1^{uu}0^{\bar{u}\bar{d}}) + 2\alpha_2^{0^{ud}0^{\bar{u}\bar{d}}} I_2(1^{u\bar{d}}0^{ud}0^{\bar{u}\bar{d}}) \\
\alpha_1^{1^{d\bar{u}}} &= \lambda + \alpha_1^{1^{\bar{u}\bar{u}}} I_1(1^{d\bar{u}}1^{\bar{u}\bar{u}}) + \alpha_1^{1^{u\bar{u}}} I_1(1^{d\bar{u}}1^{u\bar{u}}) + \alpha_1^{1^{d\bar{u}}} I_1(1^{d\bar{u}}1^{d\bar{u}}) + \alpha_1^{1^{d\bar{d}}} I_1(1^{d\bar{u}}1^{d\bar{d}}) + 2\alpha_1^{0^{ud}} I_1(1^{d\bar{u}}0^{ud}) \\
&+ \alpha_1^{0^{\bar{u}\bar{d}}} I_1(1^{d\bar{u}}0^{\bar{u}\bar{d}}) + 2\alpha_1^{0^{u\bar{u}}} I_1(1^{d\bar{u}}0^{u\bar{u}}) + \alpha_1^{0^{d\bar{u}}} I_1(1^{d\bar{u}}0^{d\bar{u}}) + \alpha_1^{0^{d\bar{d}}} I_1(1^{d\bar{u}}0^{d\bar{d}}) \\
&+ 2\alpha_2^{1^{\bar{u}\bar{u}}0^{ud}} I_2(1^{d\bar{u}}1^{\bar{u}\bar{u}}0^{ud}) + 2\alpha_2^{0^{ud}0^{\bar{u}\bar{d}}} I_2(1^{d\bar{u}}0^{ud}0^{\bar{u}\bar{d}}) \\
\alpha_1^{1^{d\bar{d}}} &= \lambda + \alpha_1^{1^{u\bar{d}}} I_1(1^{d\bar{d}}1^{u\bar{d}}) + \alpha_1^{1^{d\bar{u}}} I_1(1^{d\bar{d}}1^{d\bar{u}}) + 2\alpha_1^{0^{ud}} I_1(1^{d\bar{d}}0^{ud}) + 2\alpha_1^{0^{\bar{u}\bar{d}}} I_1(1^{d\bar{d}}0^{\bar{u}\bar{d}}) \\
&+ 2\alpha_1^{0^{u\bar{d}}} I_1(1^{d\bar{d}}0^{u\bar{d}}) + 2\alpha_1^{0^{d\bar{u}}} I_1(1^{d\bar{d}}0^{d\bar{u}}) + 2\alpha_2^{0^{ud}0^{\bar{u}\bar{d}}} I_2(1^{d\bar{d}}0^{ud}0^{\bar{u}\bar{d}}) \\
\alpha_1^{0^{ud}} &= \lambda + \alpha_1^{1^{uu}} I_1(0^{ud}1^{uu}) + 2\alpha_1^{1^{u\bar{u}}} I_1(0^{ud}1^{u\bar{u}}) + \alpha_1^{1^{u\bar{d}}} I_1(0^{ud}1^{u\bar{d}}) + 2\alpha_1^{1^{d\bar{u}}} I_1(0^{ud}1^{d\bar{u}}) + \alpha_1^{1^{d\bar{d}}} I_1(0^{ud}1^{d\bar{d}}) \\
&+ \alpha_1^{0^{ud}} I_1(0^{ud}0^{ud}) + 2\alpha_1^{0^{u\bar{u}}} I_1(0^{ud}0^{u\bar{u}}) + \alpha_1^{0^{u\bar{d}}} I_1(0^{ud}0^{u\bar{d}}) + 2\alpha_1^{0^{d\bar{u}}} I_1(0^{ud}0^{d\bar{u}}) + \alpha_1^{0^{d\bar{d}}} I_1(0^{ud}0^{d\bar{d}}) \\
\alpha_1^{0^{\bar{u}\bar{d}}} &= \lambda + \alpha_1^{1^{\bar{u}\bar{u}}} I_1(0^{\bar{u}\bar{d}}1^{\bar{u}\bar{u}}) + 2\alpha_1^{1^{u\bar{u}}} I_1(0^{\bar{u}\bar{d}}1^{u\bar{u}}) + 2\alpha_1^{1^{u\bar{d}}} I_1(0^{\bar{u}\bar{d}}1^{u\bar{d}}) + \alpha_1^{1^{d\bar{u}}} I_1(0^{\bar{u}\bar{d}}1^{d\bar{u}}) + \alpha_1^{1^{d\bar{d}}} I_1(0^{\bar{u}\bar{d}}1^{d\bar{d}}) \\
&+ \alpha_1^{0^{\bar{u}\bar{d}}} I_1(0^{\bar{u}\bar{d}}0^{\bar{u}\bar{d}}) + 2\alpha_1^{0^{u\bar{u}}} I_1(0^{\bar{u}\bar{d}}0^{u\bar{u}}) + 2\alpha_1^{0^{u\bar{d}}} I_1(0^{\bar{u}\bar{d}}0^{u\bar{d}}) + \alpha_1^{0^{d\bar{u}}} I_1(0^{\bar{u}\bar{d}}0^{d\bar{u}}) + \alpha_1^{0^{d\bar{d}}} I_1(0^{\bar{u}\bar{d}}0^{d\bar{d}})
\end{aligned}$$

$$\begin{aligned}
\alpha_1^{0^{u\bar{u}}} &= \lambda + \alpha_1^{1^{uu}} I_1(0^{u\bar{u}} 1^{uu}) + \alpha_1^{1^{\bar{u}\bar{u}}} I_1(0^{u\bar{u}} 1^{\bar{u}\bar{u}}) + 2 \alpha_1^{1^{u\bar{u}}} I_1(0^{u\bar{u}} 1^{u\bar{u}}) + \alpha_1^{1^{u\bar{d}}} I_1(0^{u\bar{u}} 1^{u\bar{d}}) + \alpha_1^{1^{d\bar{u}}} I_1(0^{u\bar{u}} 1^{d\bar{u}}) \\
&+ \alpha_1^{0^{ud}} I_1(0^{u\bar{u}} 0^{ud}) + \alpha_1^{0^{\bar{u}\bar{d}}} I_1(0^{u\bar{u}} 0^{\bar{u}\bar{d}}) + 2 \alpha_1^{0^{u\bar{u}}} I_1(0^{u\bar{u}} 0^{u\bar{u}}) + \alpha_1^{0^{u\bar{d}}} I_1(0^{u\bar{u}} 0^{u\bar{d}}) + \alpha_1^{0^{d\bar{u}}} I_1(0^{u\bar{u}} 0^{d\bar{u}}) \\
&+ \alpha_2^{1^{uu} 1^{\bar{u}\bar{u}}} I_2(0^{u\bar{u}} 1^{uu} 1^{\bar{u}\bar{u}}) + \alpha_2^{1^{uu} 0^{\bar{u}\bar{d}}} I_2(0^{u\bar{u}} 1^{uu} 0^{\bar{u}\bar{d}}) + \alpha_2^{1^{\bar{u}\bar{u}} 0^{ud}} I_2(0^{u\bar{u}} 1^{\bar{u}\bar{u}} 0^{ud}) + \alpha_2^{0^{ud} 0^{\bar{u}\bar{d}}} I_2(0^{u\bar{u}} 0^{ud} 0^{\bar{u}\bar{d}}) \\
\alpha_1^{0^{u\bar{d}}} &= \lambda + \alpha_1^{1^{uu}} I_1(0^{u\bar{d}} 1^{uu}) + 2 \alpha_1^{1^{u\bar{u}}} I_1(0^{u\bar{d}} 1^{u\bar{u}}) + \alpha_1^{1^{u\bar{d}}} I_1(0^{u\bar{d}} 1^{u\bar{d}}) + \alpha_1^{1^{d\bar{d}}} I_1(0^{u\bar{d}} 1^{d\bar{d}}) + \alpha_1^{0^{ud}} I_1(0^{u\bar{d}} 0^{ud}) \\
&+ 2 \alpha_1^{0^{\bar{u}\bar{d}}} I_1(0^{u\bar{d}} 0^{\bar{u}\bar{d}}) + 2 \alpha_1^{0^{u\bar{u}}} I_1(0^{u\bar{d}} 0^{u\bar{u}}) + \alpha_1^{0^{u\bar{d}}} I_1(0^{u\bar{d}} 0^{u\bar{d}}) + \alpha_1^{0^{d\bar{d}}} I_1(0^{u\bar{d}} 0^{d\bar{d}}) \\
&+ \alpha_2^{1^{uu} 0^{\bar{u}\bar{d}}} I_2(0^{u\bar{d}} 1^{uu} 0^{\bar{u}\bar{d}}) + 2 \alpha_2^{0^{ud} 0^{\bar{u}\bar{d}}} I_2(0^{u\bar{d}} 0^{ud} 0^{\bar{u}\bar{d}}) \\
\alpha_1^{0^{d\bar{u}}} &= \lambda + \alpha_1^{1^{\bar{u}\bar{u}}} I_1(0^{d\bar{u}} 1^{\bar{u}\bar{u}}) + 2 \alpha_1^{1^{u\bar{u}}} I_1(0^{d\bar{u}} 1^{u\bar{u}}) + \alpha_1^{1^{d\bar{u}}} I_1(0^{d\bar{u}} 1^{d\bar{u}}) + \alpha_1^{1^{d\bar{d}}} I_1(0^{d\bar{u}} 1^{d\bar{d}}) + 2 \alpha_1^{0^{ud}} I_1(0^{d\bar{u}} 0^{ud}) \\
&+ \alpha_1^{0^{\bar{u}\bar{d}}} I_1(0^{d\bar{u}} 0^{\bar{u}\bar{d}}) + 2 \alpha_1^{0^{u\bar{u}}} I_1(0^{d\bar{u}} 0^{u\bar{u}}) + \alpha_1^{0^{d\bar{u}}} I_1(0^{d\bar{u}} 0^{d\bar{u}}) + \alpha_1^{0^{d\bar{d}}} I_1(0^{d\bar{u}} 0^{d\bar{d}}) + \alpha_2^{1^{\bar{u}\bar{u}} 0^{ud}} I_2(0^{d\bar{u}} 1^{\bar{u}\bar{u}} 0^{ud}) \\
&+ 2 \alpha_2^{0^{ud} 0^{\bar{u}\bar{d}}} I_2(0^{d\bar{u}} 0^{ud} 0^{\bar{u}\bar{d}}) \\
\alpha_1^{0^{d\bar{d}}} &= \lambda + 2 \alpha_1^{1^{u\bar{d}}} I_1(0^{d\bar{d}} 1^{u\bar{d}}) + 2 \alpha_1^{1^{d\bar{u}}} I_1(0^{d\bar{d}} 1^{d\bar{u}}) + 2 \alpha_1^{0^{ud}} I_1(0^{d\bar{d}} 0^{ud}) + 2 \alpha_1^{0^{\bar{u}\bar{d}}} I_1(0^{d\bar{d}} 0^{\bar{u}\bar{d}}) \\
&+ 2 \alpha_1^{0^{u\bar{d}}} I_1(0^{d\bar{d}} 0^{u\bar{d}}) + 2 \alpha_1^{0^{d\bar{u}}} I_1(0^{d\bar{d}} 0^{d\bar{u}}) + 4 \alpha_2^{0^{ud} 0^{\bar{u}\bar{d}}} I_2(0^{d\bar{d}} 0^{ud} 0^{\bar{u}\bar{d}}) \\
\alpha_2^{1^{uu} 1^{\bar{u}\bar{u}}} &= \lambda + 2 \alpha_1^{0^{ud}} I_4(1^{uu} 1^{\bar{u}\bar{u}} 0^{ud}) + 2 \alpha_1^{0^{\bar{u}\bar{d}}} I_4(1^{\bar{u}\bar{u}} 1^{uu} 0^{\bar{u}\bar{d}}) + 4 \alpha_1^{0^{u\bar{u}}} I_3(1^{uu} 1^{\bar{u}\bar{u}} 0^{u\bar{u}}) \\
&+ 4 \alpha_2^{0^{ud} 0^{\bar{u}\bar{d}}} I_6(1^{uu} 1^{\bar{u}\bar{u}} 0^{ud} 0^{\bar{u}\bar{d}}) + 4 \alpha_3^{0^{ud} 0^{u\bar{u}} 0^{\bar{u}\bar{d}}} I_8(1^{uu} 1^{\bar{u}\bar{u}} 0^{ud} 0^{u\bar{u}} 0^{\bar{u}\bar{d}}) \\
\alpha_2^{1^{uu} 0^{\bar{u}\bar{d}}} &= \lambda + \alpha_1^{1^{\bar{u}\bar{u}}} I_4(0^{\bar{u}\bar{d}} 1^{uu} 1^{\bar{u}\bar{u}}) + 2 \alpha_1^{1^{u\bar{u}}} I_3(1^{uu} 0^{\bar{u}\bar{d}} 1^{u\bar{u}}) + 2 \alpha_1^{1^{u\bar{d}}} I_3(1^{uu} 0^{\bar{u}\bar{d}} 1^{u\bar{d}}) + 2 \alpha_1^{0^{ud}} I_4(1^{uu} 0^{\bar{u}\bar{d}} 0^{ud}) \\
&+ \alpha_1^{0^{\bar{u}\bar{d}}} I_4(0^{\bar{u}\bar{d}} 1^{uu} 0^{\bar{u}\bar{d}}) + 2 \alpha_1^{0^{u\bar{u}}} I_3(1^{uu} 0^{\bar{u}\bar{d}} 0^{u\bar{u}}) + 2 \alpha_1^{0^{u\bar{d}}} I_3(1^{uu} 0^{\bar{u}\bar{d}} 0^{u\bar{d}}) + 2 \alpha_2^{1^{\bar{u}\bar{u}} 0^{ud}} I_6(1^{uu} 0^{\bar{u}\bar{d}} 0^{ud} 1^{\bar{u}\bar{u}}) \\
&+ 2 \alpha_2^{0^{ud} 0^{\bar{u}\bar{d}}} I_6(1^{uu} 0^{\bar{u}\bar{d}} 0^{ud} 0^{\bar{u}\bar{d}}) + 2 \alpha_3^{1^{\bar{u}\bar{u}} 1^{u\bar{d}} 0^{ud}} I_8(1^{uu} 0^{\bar{u}\bar{d}} 0^{ud} 1^{u\bar{d}} 1^{\bar{u}\bar{u}}) \\
&+ 2 \alpha_3^{0^{ud} 0^{u\bar{u}} 0^{\bar{u}\bar{d}}} I_8(1^{uu} 0^{\bar{u}\bar{d}} 0^{ud} 0^{u\bar{u}} 0^{\bar{u}\bar{d}}) \\
\alpha_2^{1^{\bar{u}\bar{u}} 0^{ud}} &= \lambda + \alpha_1^{1^{uu}} I_4(0^{ud} 1^{\bar{u}\bar{u}} 1^{uu}) + 2 \alpha_1^{1^{u\bar{u}}} I_3(1^{\bar{u}\bar{u}} 0^{ud} 1^{u\bar{u}}) + 2 \alpha_1^{1^{d\bar{u}}} I_3(1^{\bar{u}\bar{u}} 0^{ud} 1^{d\bar{u}}) + \alpha_1^{0^{ud}} I_4(0^{ud} 1^{\bar{u}\bar{u}} 0^{ud}) \\
&+ 2 \alpha_1^{0^{\bar{u}\bar{d}}} I_4(1^{\bar{u}\bar{u}} 0^{ud} 0^{\bar{u}\bar{d}}) + 2 \alpha_1^{0^{u\bar{u}}} I_3(1^{\bar{u}\bar{u}} 0^{ud} 0^{u\bar{u}}) + 2 \alpha_1^{0^{d\bar{u}}} I_3(1^{\bar{u}\bar{u}} 0^{ud} 0^{d\bar{u}}) \\
&+ 2 \alpha_2^{1^{uu} 0^{\bar{u}\bar{d}}} I_6(1^{\bar{u}\bar{u}} 0^{ud} 0^{\bar{u}\bar{d}} 1^{uu}) + 2 \alpha_2^{0^{ud} 0^{\bar{u}\bar{d}}} I_6(1^{\bar{u}\bar{u}} 0^{ud} 0^{\bar{u}\bar{d}} 0^{ud}) + 2 \alpha_3^{1^{uu} 1^{d\bar{u}} 0^{\bar{u}\bar{d}}} I_8(1^{\bar{u}\bar{u}} 0^{ud} 0^{\bar{u}\bar{d}} 1^{d\bar{u}} 1^{uu}) \\
&+ 2 \alpha_3^{0^{ud} 0^{u\bar{u}} 0^{\bar{u}\bar{d}}} I_8(1^{\bar{u}\bar{u}} 0^{ud} 0^{\bar{u}\bar{d}} 0^{u\bar{u}} 0^{ud})
\end{aligned}$$

$$\begin{aligned}
\alpha_2^{0^{ud}0^{\bar{u}\bar{d}}} &= \lambda + \alpha_1^{1^{uu}} I_4(0^{ud}0^{\bar{u}\bar{d}}1^{uu}) + \alpha_1^{1^{\bar{u}\bar{u}}} I_4(0^{\bar{u}\bar{d}}0^{ud}1^{\bar{u}\bar{u}}) + \alpha_1^{1^{u\bar{u}}} I_3(0^{ud}0^{\bar{u}\bar{d}}1^{u\bar{u}}) + \alpha_1^{1^{u\bar{d}}} I_3(0^{ud}0^{\bar{u}\bar{d}}1^{u\bar{d}}) \\
&+ \alpha_1^{1^{d\bar{u}}} I_3(0^{ud}0^{\bar{u}\bar{d}}1^{d\bar{u}}) + \alpha_1^{1^{d\bar{d}}} I_3(0^{ud}0^{\bar{u}\bar{d}}1^{d\bar{d}}) + \alpha_1^{0^{ud}} I_4(0^{ud}0^{\bar{u}\bar{d}}0^{ud}) + \alpha_1^{0^{\bar{u}\bar{d}}} I_4(0^{\bar{u}\bar{d}}0^{ud}0^{\bar{u}\bar{d}}) \\
&+ \alpha_1^{0^{u\bar{u}}} I_3(0^{ud}0^{\bar{u}\bar{d}}0^{u\bar{u}}) + \alpha_1^{0^{u\bar{d}}} I_3(0^{ud}0^{\bar{u}\bar{d}}0^{u\bar{d}}) + \alpha_1^{0^{d\bar{u}}} I_3(0^{ud}0^{\bar{u}\bar{d}}0^{d\bar{u}}) + \alpha_1^{0^{d\bar{d}}} I_3(0^{ud}0^{\bar{u}\bar{d}}0^{d\bar{d}}) \\
&+ \alpha_2^{1^{uu}1^{\bar{u}\bar{u}}} I_6(0^{ud}0^{\bar{u}\bar{d}}1^{uu}1^{\bar{u}\bar{u}}) + \alpha_2^{1^{uu}0^{\bar{u}\bar{d}}} I_6(0^{ud}0^{\bar{u}\bar{d}}1^{uu}0^{\bar{u}\bar{d}}) + \alpha_2^{1^{\bar{u}\bar{u}}0^{ud}} I_6(0^{ud}0^{\bar{u}\bar{d}}0^{ud}1^{\bar{u}\bar{u}}) \\
&+ \alpha_2^{0^{ud}0^{\bar{u}\bar{d}}} I_6(0^{ud}0^{\bar{u}\bar{d}}0^{ud}0^{\bar{u}\bar{d}}) + \alpha_3^{1^{uu}1^{\bar{u}\bar{u}}0^{d\bar{d}}} I_8(0^{ud}0^{\bar{u}\bar{d}}1^{uu}0^{d\bar{d}}1^{\bar{u}\bar{u}}) + \alpha_3^{1^{uu}0^{d\bar{u}}0^{\bar{u}\bar{d}}} I_8(0^{ud}0^{\bar{u}\bar{d}}1^{uu}0^{d\bar{u}}0^{\bar{u}\bar{d}}) \\
&+ \alpha_3^{1^{\bar{u}\bar{u}}1^{u\bar{d}}0^{ud}} I_8(0^{ud}0^{\bar{u}\bar{d}}0^{ud}1^{u\bar{d}}1^{\bar{u}\bar{u}}) + \alpha_3^{0^{ud}0^{u\bar{u}}0^{\bar{u}\bar{d}}} I_8(0^{ud}0^{\bar{u}\bar{d}}0^{ud}0^{u\bar{u}}0^{\bar{u}\bar{d}}) \\
\alpha_3^{1^{uu}1^{\bar{u}\bar{u}}0^{d\bar{d}}} &= \lambda + 2\alpha_1^{1^{u\bar{d}}} I_9(1^{uu}0^{d\bar{d}}1^{\bar{u}\bar{u}}1^{u\bar{d}}) + 2\alpha_1^{1^{d\bar{u}}} I_9(1^{\bar{u}\bar{u}}0^{d\bar{d}}1^{uu}1^{d\bar{u}}) + 2\alpha_1^{0^{ud}} I_9(1^{uu}0^{d\bar{d}}1^{\bar{u}\bar{u}}0^{ud}) \\
&+ 2\alpha_1^{0^{\bar{u}\bar{d}}} I_9(1^{\bar{u}\bar{u}}0^{d\bar{d}}1^{uu}0^{\bar{u}\bar{d}}) + 4\alpha_1^{0^{u\bar{u}}} I_9(1^{uu}1^{\bar{u}\bar{u}}0^{d\bar{d}}0^{u\bar{u}}) + 2\alpha_1^{0^{u\bar{d}}} I_9(1^{uu}0^{d\bar{d}}1^{\bar{u}\bar{u}}0^{u\bar{d}}) \\
&+ 2\alpha_1^{0^{d\bar{u}}} I_9(1^{\bar{u}\bar{u}}0^{d\bar{d}}1^{uu}0^{d\bar{u}}) + 4\alpha_2^{0^{ud}0^{\bar{u}\bar{d}}} I_{10}(1^{uu}0^{d\bar{d}}1^{\bar{u}\bar{u}}0^{ud}0^{\bar{u}\bar{d}}) \\
\alpha_3^{1^{uu}1^{d\bar{u}}0^{\bar{u}\bar{d}}} &= \lambda + \alpha_1^{1^{\bar{u}\bar{u}}} I_9(1^{d\bar{u}}0^{\bar{u}\bar{d}}1^{uu}1^{\bar{u}\bar{u}}) + 2\alpha_1^{1^{u\bar{u}}} I_9(1^{uu}0^{\bar{u}\bar{d}}1^{d\bar{u}}1^{u\bar{u}}) + 2\alpha_1^{1^{u\bar{d}}} I_9(1^{uu}0^{\bar{u}\bar{d}}1^{d\bar{u}}1^{u\bar{d}}) \\
&+ \alpha_1^{1^{d\bar{u}}} I_9(1^{d\bar{u}}0^{\bar{u}\bar{d}}1^{uu}1^{d\bar{u}}) + \alpha_1^{1^{d\bar{d}}} I_9(1^{d\bar{u}}0^{\bar{u}\bar{d}}1^{uu}1^{d\bar{d}}) + 2\alpha_1^{0^{ud}} I_9(1^{uu}1^{d\bar{u}}0^{\bar{u}\bar{d}}0^{ud}) \\
&+ \alpha_1^{0^{\bar{u}\bar{d}}} I_9(1^{d\bar{u}}0^{\bar{u}\bar{d}}1^{uu}0^{\bar{u}\bar{d}}) + \alpha_1^{0^{u\bar{u}}} (2I_9(1^{uu}1^{d\bar{u}}0^{\bar{u}\bar{d}}0^{u\bar{u}}) + 2I_9(1^{uu}0^{\bar{u}\bar{d}}1^{d\bar{u}}0^{u\bar{u}})) \\
&+ 2\alpha_1^{0^{u\bar{d}}} I_9(1^{uu}0^{\bar{u}\bar{d}}1^{d\bar{u}}0^{u\bar{d}}) + \alpha_1^{0^{d\bar{u}}} I_9(1^{d\bar{u}}0^{\bar{u}\bar{d}}1^{uu}0^{d\bar{u}}) + \alpha_1^{0^{d\bar{d}}} I_9(1^{d\bar{u}}0^{\bar{u}\bar{d}}1^{uu}0^{d\bar{d}}) \\
&+ 2\alpha_2^{1^{\bar{u}\bar{u}}0^{ud}} I_{10}(1^{uu}1^{d\bar{u}}0^{\bar{u}\bar{d}}0^{ud}1^{\bar{u}\bar{u}}) + 2\alpha_2^{0^{ud}0^{\bar{u}\bar{d}}} I_{10}(1^{uu}1^{d\bar{u}}0^{\bar{u}\bar{d}}0^{ud}0^{\bar{u}\bar{d}}) \\
\alpha_3^{1^{\bar{u}\bar{u}}1^{u\bar{d}}0^{ud}} &= \lambda + \alpha_1^{1^{uu}} I_9(1^{u\bar{d}}0^{ud}1^{\bar{u}\bar{u}}1^{uu}) + 2\alpha_1^{1^{u\bar{u}}} I_9(1^{\bar{u}\bar{u}}0^{ud}1^{u\bar{d}}1^{u\bar{u}}) + \alpha_1^{1^{u\bar{d}}} I_9(1^{u\bar{d}}0^{ud}1^{\bar{u}\bar{u}}1^{u\bar{d}}) \\
&+ 2\alpha_1^{1^{d\bar{u}}} I_9(1^{\bar{u}\bar{u}}0^{ud}1^{u\bar{d}}1^{d\bar{u}}) + \alpha_1^{1^{d\bar{d}}} I_9(1^{u\bar{d}}0^{ud}1^{\bar{u}\bar{u}}1^{d\bar{d}}) + \alpha_1^{0^{ud}} I_9(1^{u\bar{d}}0^{ud}1^{\bar{u}\bar{u}}0^{ud}) \\
&+ 2\alpha_1^{0^{\bar{u}\bar{d}}} I_9(1^{\bar{u}\bar{u}}1^{u\bar{d}}0^{ud}0^{\bar{u}\bar{d}}) + \alpha_1^{0^{u\bar{u}}} (2I_9(1^{\bar{u}\bar{u}}1^{u\bar{d}}0^{ud}0^{u\bar{u}}) + 2I_9(1^{\bar{u}\bar{u}}0^{ud}1^{u\bar{d}}0^{u\bar{u}})) \\
&+ \alpha_1^{0^{u\bar{d}}} I_9(1^{u\bar{d}}0^{ud}1^{\bar{u}\bar{u}}0^{u\bar{d}}) + 2\alpha_1^{0^{d\bar{u}}} I_9(1^{\bar{u}\bar{u}}0^{ud}1^{u\bar{d}}0^{d\bar{u}}) + \alpha_1^{0^{d\bar{d}}} I_9(1^{u\bar{d}}0^{ud}1^{\bar{u}\bar{u}}0^{d\bar{d}}) \\
&+ 2\alpha_2^{1^{uu}0^{\bar{u}\bar{d}}} I_{10}(1^{\bar{u}\bar{u}}1^{u\bar{d}}0^{ud}0^{\bar{u}\bar{d}}1^{uu}) + 2\alpha_2^{0^{ud}0^{\bar{u}\bar{d}}} I_{10}(1^{\bar{u}\bar{u}}1^{u\bar{d}}0^{ud}0^{\bar{u}\bar{d}}0^{ud}) \\
\alpha_3^{0^{ud}0^{u\bar{u}}0^{\bar{u}\bar{d}}} &= \lambda + \alpha_1^{1^{uu}} I_9(0^{ud}0^{u\bar{u}}0^{\bar{u}\bar{d}}1^{uu}) + \alpha_1^{1^{\bar{u}\bar{u}}} I_9(0^{\bar{u}\bar{d}}0^{u\bar{u}}0^{ud}1^{\bar{u}\bar{u}}) + \alpha_1^{1^{u\bar{u}}} (I_9(0^{ud}0^{u\bar{u}}0^{\bar{u}\bar{d}}1^{u\bar{u}})
\end{aligned}$$

$$\begin{aligned}
& + I_9(0^{\bar{u}\bar{d}}0^{u\bar{u}}0^{ud}1^{u\bar{u}}) + I_9(0^{ud}0^{\bar{u}\bar{d}}0^{u\bar{u}}1^{u\bar{u}}) + \alpha_1^{1^{u\bar{d}}}(I_9(0^{\bar{u}\bar{d}}0^{u\bar{u}}0^{ud}1^{u\bar{d}}) + I_9(0^{ud}0^{\bar{u}\bar{d}}0^{u\bar{u}}1^{u\bar{d}})) \\
& + \alpha_1^{1^{d\bar{u}}}(I_9(0^{ud}0^{u\bar{u}}0^{\bar{u}\bar{d}}1^{d\bar{u}}) + I_9(0^{ud}0^{\bar{u}\bar{d}}0^{u\bar{u}}1^{d\bar{u}})) + \alpha_1^{1^{d\bar{d}}}(I_9(0^{ud}0^{\bar{u}\bar{d}}0^{u\bar{u}}1^{d\bar{d}}) + \alpha_1^{0^{ud}}I_9(0^{ud}0^{u\bar{u}}0^{\bar{u}\bar{d}}0^{ud})) \\
& + \alpha_1^{0^{\bar{u}\bar{d}}}(I_9(0^{\bar{u}\bar{d}}0^{u\bar{u}}0^{ud}0^{\bar{u}\bar{d}}) + \alpha_1^{0^{u\bar{u}}}(I_9(0^{ud}0^{u\bar{u}}0^{\bar{u}\bar{d}}0^{u\bar{u}}) + I_9(0^{\bar{u}\bar{d}}0^{u\bar{u}}0^{ud}0^{u\bar{u}}) + I_9(0^{ud}0^{\bar{u}\bar{d}}0^{u\bar{u}}0^{u\bar{u}})) \\
& + \alpha_1^{0^{u\bar{d}}}(I_9(0^{\bar{u}\bar{d}}0^{u\bar{u}}0^{ud}0^{u\bar{d}}) + I_9(0^{ud}0^{\bar{u}\bar{d}}0^{u\bar{u}}0^{u\bar{d}})) + \alpha_1^{0^{d\bar{u}}}(I_9(0^{ud}0^{u\bar{u}}0^{\bar{u}\bar{d}}0^{d\bar{u}}) + I_9(0^{ud}0^{\bar{u}\bar{d}}0^{u\bar{u}}0^{d\bar{u}})) \\
& + \alpha_1^{0^{d\bar{d}}}(I_9(0^{ud}0^{\bar{u}\bar{d}}0^{u\bar{u}}0^{d\bar{d}}) + \alpha_2^{1^{uu}1^{\bar{u}\bar{u}}}I_{10}(0^{ud}0^{u\bar{u}}0^{\bar{u}\bar{d}}1^{uu}1^{\bar{u}\bar{u}}) + \alpha_2^{1^{uu}0^{\bar{u}\bar{d}}}I_{10}(0^{ud}0^{u\bar{u}}0^{\bar{u}\bar{d}}1^{uu}0^{\bar{u}\bar{d}}) \\
& + \alpha_2^{1^{\bar{u}\bar{u}}0^{ud}}I_{10}(0^{ud}0^{u\bar{u}}0^{\bar{u}\bar{d}}0^{ud}1^{\bar{u}\bar{u}}) + \alpha_2^{0^{ud}0^{\bar{u}\bar{d}}}I_{10}(0^{ud}0^{u\bar{u}}0^{\bar{u}\bar{d}}0^{ud}0^{\bar{u}\bar{d}})
\end{aligned}$$

Table I. S -wave baryon masses $M(J^p)$ (GeV).

$M(\frac{1}{2}^+)$			$M(\frac{3}{2}^+)$		
N	0.940	(0.940)	Δ	1.232	(1.232)
	0.940			1.232	
Λ	1.022	(1.116)	Σ^*	1.377	(1.385)
	1.098			1.377	
Σ	1.050	(1.193)	Ξ^*	1.524	(1.530)
	1.193			1.524	
Ξ	1.162	(1.315)	Ω	1.672	(1.672)
	1.325			1.672	

Table II. S -wave baryonia masses. Parameters of model: cutoff $\Lambda = 11.0$, gluon coupling constant $g = 0.314$. Quark masses $m_{u,d} = 410 MeV$.

I	Quark content (baryonia)	J	Mass (MeV)
0	$uuu\bar{u}\bar{u}\bar{u}$ ($\Delta\bar{\Delta}$), $ddd\bar{d}\bar{d}\bar{d}$ ($\Delta\bar{\Delta}$)	0	1973
		1	1824
		2	1938
		3	2290
	$udd\bar{u}\bar{d}\bar{d}$ ($\Delta\bar{\Delta} + \Delta\bar{n} + n\bar{\Delta} + n\bar{n}$), $uud\bar{u}\bar{u}\bar{d}$ ($\Delta\bar{\Delta} + \Delta\bar{p} + p\bar{\Delta} + p\bar{p}$)	0	1835
		1	1784
		2	1851
		3	2455
1	$udd\bar{d}\bar{d}\bar{d}$ ($\Delta\bar{\Delta} + n\bar{\Delta}$), $uuu\bar{u}\bar{u}\bar{d}$ ($\Delta\bar{\Delta} + \Delta\bar{p}$)	0	1928
		1	1770
		2	1857
		3	2395
	$uud\bar{u}\bar{d}\bar{d}$ ($\Delta\bar{\Delta} + \Delta\bar{n} + p\bar{\Delta} + p\bar{n}$)	0	1835
		1	1784
		2	1851
		3	2455
2	$uudd\bar{d}\bar{d}\bar{d}$ ($\Delta\bar{\Delta} + p\bar{\Delta}$), $uuu\bar{u}\bar{d}\bar{d}\bar{d}$ ($\Delta\bar{\Delta} + \Delta\bar{n}$)	0	1928
		1	1770
		2	1857
		3	2395
3	$uuu\bar{d}\bar{d}\bar{d}\bar{d}$ ($\Delta\bar{\Delta}$)	0	2067
		1	1783
		2	1938
		3	2290

Table III. $IJ = 00$, $uud\bar{u}\bar{d}$ (1835 MeV), $\Lambda = 11.0$, $g = 0.314$.

Subamplitudes	Contributions, percent
A_1^{1uu}	3.7
$A_1^{1\bar{u}\bar{u}}$	3.7
$A_1^{1u\bar{u}}$	9.0
$A_1^{1u\bar{d}}$	7.8
$A_1^{1d\bar{u}}$	7.8
$A_1^{1d\bar{d}}$	6.7
A_1^{0ud}	3.6
$A_1^{0\bar{u}\bar{d}}$	3.6
$A_1^{0u\bar{u}}$	7.0
$A_1^{0u\bar{d}}$	6.8
$A_1^{0d\bar{u}}$	6.8
$A_1^{0d\bar{d}}$	6.6
$A_2^{1uu1\bar{u}\bar{u}}$	2.4
$A_2^{1uu0\bar{u}\bar{d}}$	2.0
$A_2^{1\bar{u}\bar{u}0ud}$	2.6
$A_2^{0ud0\bar{u}\bar{d}}$	2.8
$A_3^{1uu0d\bar{d}1\bar{u}\bar{u}}$	4.0
$A_3^{1uu1d\bar{u}0\bar{u}\bar{d}}$	4.5
$A_3^{1\bar{u}\bar{u}1u\bar{d}0ud}$	4.5
$A_3^{0ud0u\bar{u}0\bar{u}\bar{d}}$	4.3
$\sum A_1$	73.0
$\sum A_2$	9.8
$\sum A_3$	17.2

Table IV. $IJ = 33$, $uuu\bar{d}\bar{d}\bar{d}$ (2290 MeV), $\Lambda = 11.0$, $g = 0.314$.

Subamplitudes	Contributions, percent
A_1^{1uu}	9.9
$A_1^{1\bar{d}\bar{d}}$	9.9
$A_1^{1u\bar{d}}$	25.4
$A_2^{1uu1\bar{d}\bar{d}}$	14.5
$A_3^{1uu1\bar{d}\bar{d}1u\bar{d}}$	40.3
$\sum A_1$	45.2
$\sum A_2$	14.5
$\sum A_3$	40.3

Table V. Vertex functions and Chew-Mandelstam coefficients.

i	$G_i^2(s_{kl})$	α_i	β_i	δ_i
0^+ diquark	$\frac{4g}{3} - \frac{8gm_{kl}^2}{(3s_{kl})}$	$\frac{1}{2}$	$-\frac{1}{2} \frac{(m_k - m_l)^2}{(m_k + m_l)^2}$	0
1^+ diquark	$\frac{2g}{3}$	$\frac{1}{3}$	$\frac{4m_k m_l}{3(m_k + m_l)^2} - \frac{1}{6}$	$-\frac{1}{6} \frac{(m_k - m_l)^2}{(m_k + m_l)^2}$
0^- meson	$\frac{8g}{3} - \frac{16gm_{kl}^2}{(3s_{kl})}$	$\frac{1}{2}$	$-\frac{1}{2} \frac{(m_k - m_l)^2}{(m_k + m_l)^2}$	0
1^- meson	$\frac{4g}{3}$	$\frac{1}{3}$	$\frac{4m_k m_l}{3(m_k + m_l)^2} - \frac{1}{6}$	$-\frac{1}{6} \frac{(m_k - m_l)^2}{(m_k + m_l)^2}$

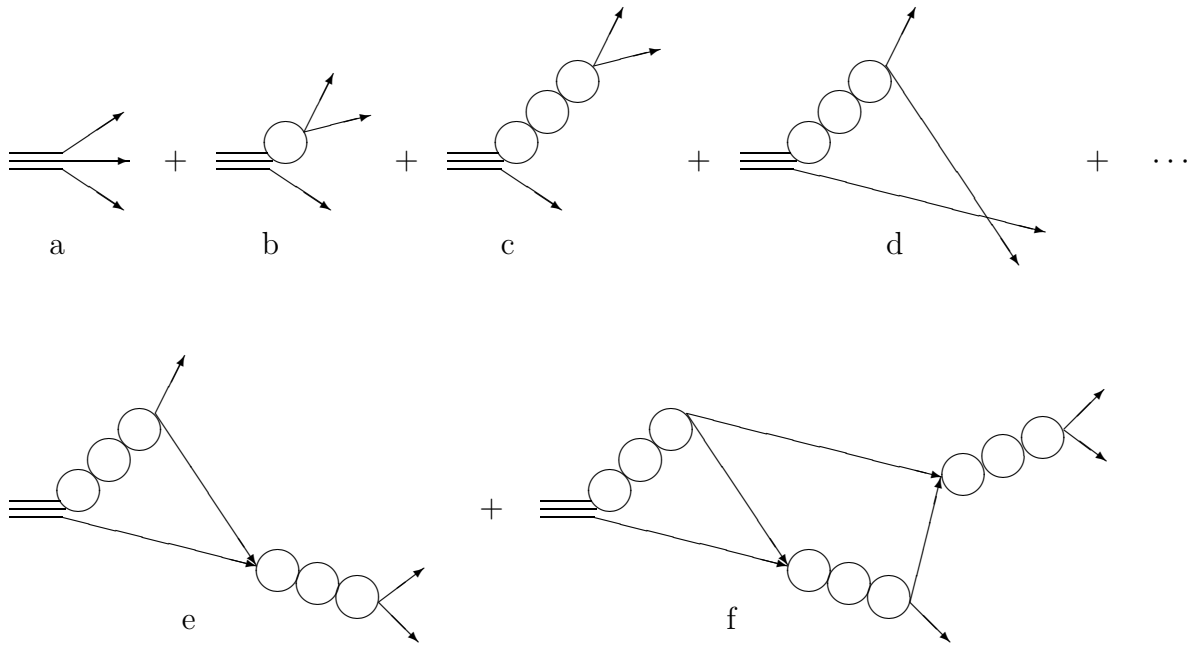


Fig. 1. Diagrams which correspond to a) production of three quarks, b – f) subsequent pair interaction.

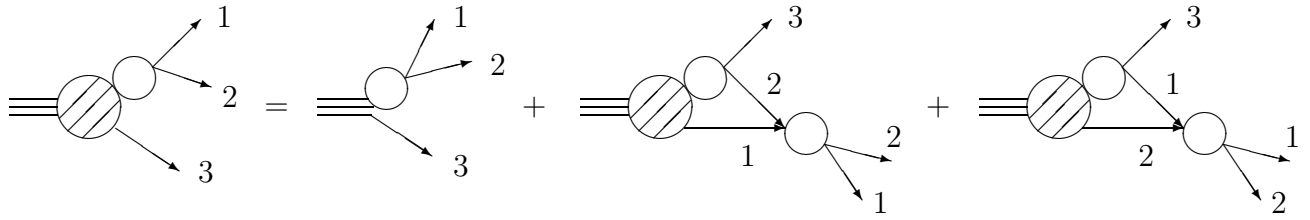


Fig. 2. Graphic representation of the equation for the amplitude $A_1(s, s_{12})$.

$$\begin{aligned}
& \text{Diagram 1} = \text{Diagram 2} + 4 \text{Diagram 3} + 2 \text{Diagram 4} \\
& + 2 \text{Diagram 5} + 4 \text{Diagram 6} \\
& + 4 \text{Diagram 7}
\end{aligned}$$

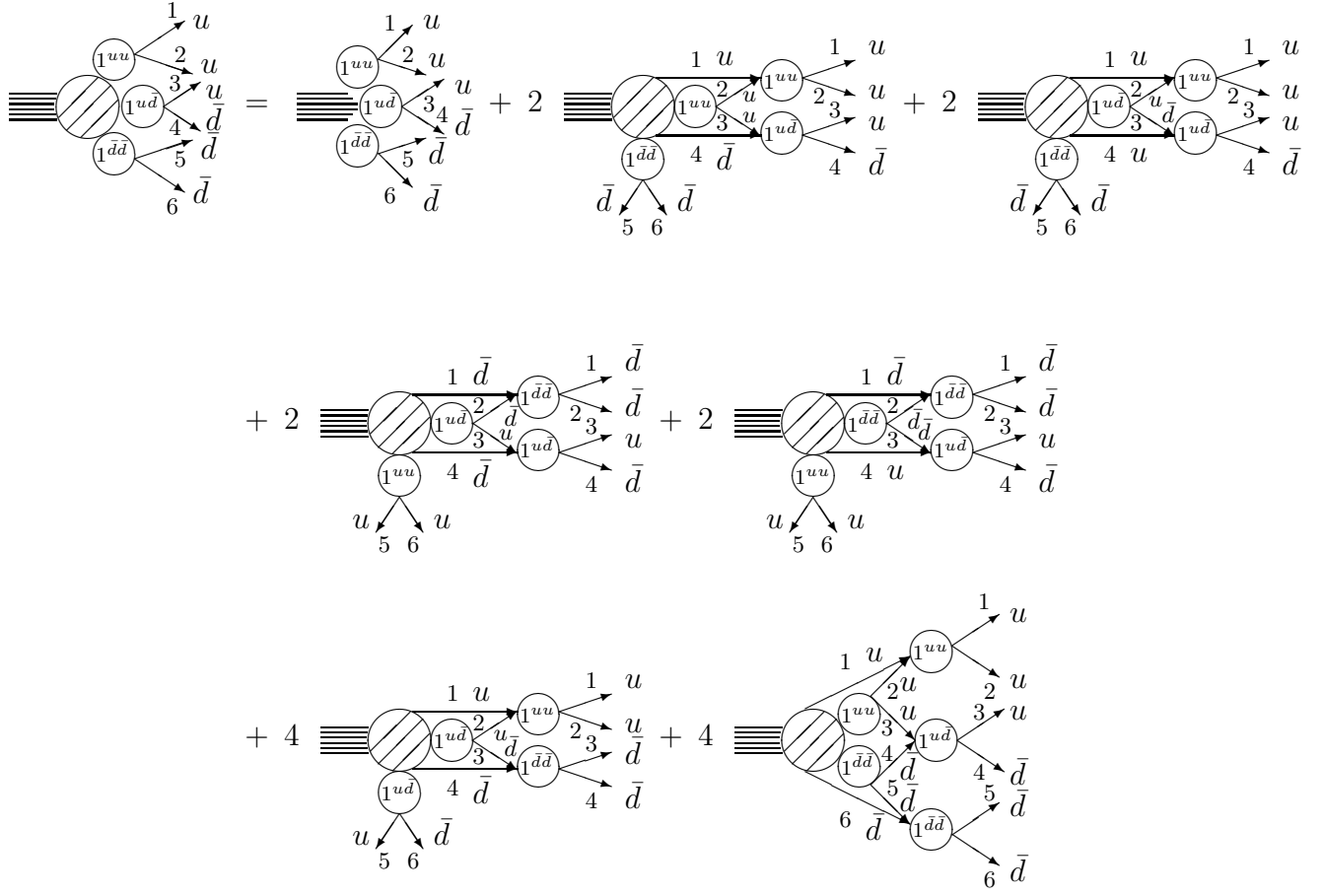


Fig. 3. Graphic representation of the equations for the six-quark subamplitudes A_l ($l = 1, 2, 3$) in the case of baryonium $uuud\bar{d}\bar{d}$ $IJ = 33$.